Wireless Network Pricing
Chapter 7: Network Externalities

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The Book

- E-Book freely downloadable from NCEL website: http://ncel.ie.cuhk.edu.hk/content/wireless-network-pricing
- Physical book available for purchase from Morgan & Claypool (http://goo.gl/JFGlai) and Amazon (http://goo.gl/JQKaEq)
Chapter 7: Network Externalities
Section 7.1: Theory: Network Externalities
What is Externality?

Definition (Externality)

An externality is any side effect (benefit or cost) that is imposed by the actions of a player on a third-party not directly involved.
Examples: Negative Externality

Air Pollution (source: Internet)
Examples: Negative Externality

Second-hand Smoke (source: Internet)
Examples: Negative Externality

Traffic Congestion (source: Internet)
Examples: Positive Externality

Lighthouse (source: Internet)
Examples: Positive Externality

Bee Keeping (source: Internet)
Examples: Positive Externality

Immunization (source: Internet)
Impact of Externality

- Can cause **market failure** without proper prices
  - The market outcome will no longer be efficient.
  - If market prices do not reflect the costs or benefits of externalities.

- Example: negative externality of pollution
  - The market price for steel reflects the cost labor, capital, and other inputs, but may not include the cost due to air pollution.
  - The steel manufacturer may produce more products than the socially optimal level.
Graphical Illustration of Market Failure

- **Social optimal production level** $Q^*$:
  - Social Marginal Cost (MC) = Social Marginal Revenue (MR)

- **Left**: negative production externality
  - Private MC < Social MC
  - Local optimal quality $Q_1 >$ Social optimal quality $Q^*$

- **Right**: positive consumption externality
  - Private MR < Social MR
  - Local optimal quality $Q_1 <$ Social optimal quality $Q^*$
Negative Network Externality
A Case Study: Water Pollution

- The chemical company produces chemical products and discharges wastewater into the river.
- The water company produces bottle water by drawing water from the river.
- Water pollution increases the production cost of the water company.
Constant MR per chemical product: $10.

Social MC = private MC (chemical plant) + external MC (pollution)

Social optimal quant $Q^*$ < local optimal quality $Q_1$
At Local Optimal Quality $Q_1$

- The chemical plant’s profit (i.e., revenue - cost):
  \[ \int_{0}^{Q_1} (MR - MC_{Private}(Q)) \, dQ = A + B + E \]

- The water company’s profit due to externality (assuming 0 revenue):
  \[ -\int_{0}^{Q_1} MC_{External}(Q) \, dQ = -(C + F) \]

- Since $C = B$ and $F = D + E$, the social surplus (sum of two profits):
  \[ A + B + E - (C + F) = A - D \]
At Social Optimal Quality $Q^*$

- The chemical plant’s profit (i.e., revenue - cost):\
  \[
  \int_0^{Q^*} (MR - MC_{Private}(Q)) \, dQ = A + B
  \]

- The water company’s profit due to externality (assuming 0 revenue):\
  \[
  - \int_0^{Q^*} MC_{External}(Q) \, dQ = -C
  \]

- Since $C = B$, the social surplus (sum of two profits):\
  \[
  A + B - C = A
  \]
Comparison

- Social surplus at $Q_1 : A - D$
- Social surplus at $Q^* : A$
- With negative externally, individual profit maximization hurts the social surplus
- Solution: Pigovian tax
Pigovian Tax

- Charge chemical plant a tax
  - Tax = external marginal cost at the optimal solution $Q^*$

- Individual profit maximisation leads to production level of $Q^*$
  - Chemical plant profit = $\int_0^{Q^*} (MR - MC_{Private}(Q) - Tax) \, dQ = A_1$
The Coase Theorem

- Nobel Laureate Ronald Coase proposes another view of externality
- Assumptions: Transaction cost is negligible, property rights are clear
- Result: Trade in externality will lead to efficient use of the resource
- Back to the previous example
  - If water company owns the water: it can charge the chemical plant a price equal to the negative externally
  - If chemical plant owns the water: it can demand a compensation from water company for reducing the chemical production quantity
  - Either way, it is possible to maximize social surplus
Positive Network Externality
A Case Study: Network Effect

- More usage of the product by any user increases the product’s value for other users.
Metcalfe’s Law

Consider a network of $N$ users.

Each user perceives a value increasing in $N$.

Each user attaches the same value to the possibility of connecting with any one of the other $N-1$ users.

Total network value $N(N-1) \approx N^2$. 

Briscore’s Refinement

- Each user ranks other users in terms of decreasing importance.
- Attach a value of $1/k$ to the $k_{th}$ important neighbour.
- Total network value $N \left( \sum_{k=1}^{N-1} \frac{1}{k} \right) \approx N \log N$. 
Different Types of Network Effect

- Direct network effect: telephone, online social network
- Indirect network effect: Office for Windows, DVDs for DVD players
- Local network effect: instant messaging
Section 7.2: Distributed Wireless Interference Compensation
Wireless Power Control

- **Distributed** power control in wireless ad hoc networks
- **Elastic** applications with no SINR targets
- Want to **maximize the total network performance**
Network Model

- **Single-hop** transmissions.
- A user = a transmitter/receiver pair.
- Transmit over one or multiple parallel channels.
- Interferences in the same channel (**negative externality**).
We focus on a single channel.

For each user $n \in \mathcal{N} = \{1, \ldots, N\}$:

- Power constraint: $p_n \in [P_n^{\text{min}}, P_n^{\text{max}}]$.
- Received SINR (signal-to-interference plus noise ratio):
  \[\gamma_n = \frac{p_n h_{n,n}}{\sigma_n + \sum_{m \neq n} p_m h_{n,m}}.\]
- Utility function $U_n(\gamma_n)$: increasing, differentiable, strictly concave.
Network Utility Maximization (NUM) Problem

**NUM Problem**

\[
\max_{\{P_n^{\text{min}} \leq p_n \leq P_n^{\text{max}}, \forall n\}} \sum_n U_n(\gamma_n).
\]

- **Technical Challenges:**
  - Coupled across users due to interferences.
  - Could be non-convex in power (check the Hessian matrix).

- **We want:** efficient and distributed algorithm, with limited information exchange and fast convergence.
Benchmark - No Information Exchange

- Each user picks power to maximize its own utility, given current interference and channel gain.

- Results in $p_n = P_n^{\text{max}}$ for all $n$.
  - Maximum interference.
  - Can be far from optimal.
Benchmark - No Information Exchange

- Each user picks power to maximize its own utility, given current interference and channel gain.

- Results in $p_n = P_{n}^{\text{max}}$ for all $n$.
  - Maximum interference.
  - Can be far from optimal.

- We propose algorithm with limited information exchange.
  - Have nice interpretation as distributed Pigovian taxation.
  - Analyze its behavior using supermodular game theory.
ADP Algorithm: Asynchronous Distributed Pricing

- **Price Announcing**: user $n$ announces “price” (per unit interference):

  $$\pi_n = \left| \frac{\partial U_n(\gamma_n)}{\partial I_n} \right| = \frac{\partial U_n(\gamma_n)}{\partial \gamma_n} \frac{\gamma_n^2}{p_n h_{n,n}}.$$  

- **Power Updating**: user $n$ updates power $p_n$ to maximize surplus:

  $$S_n = U_n(\gamma_n) - p_n \sum_{m \neq n} \pi_m h_{m,n}.$$  

- Repeat two phases **asynchronously** across users.

- **Scalable and distributed**: only need to announce single price, and know limited channel gains ($h_{m,n}$).
ADP Algorithm

- Interpretation of prices: Pigovian taxation
ADP Algorithm

- Interpretation of prices: Pigovian taxation

- ADP algorithm: distributed discovery of Pigovian taxes
  - When does it converge?
  - What does it converge to?
  - Will it solve NUM Problem?
  - How fast does it converge?
Convergence

- Depends on the utility functions.
Convergence

- Depends on the utility functions.
- Coefficient of relative Risk Aversion (CRA) of $U(\gamma)$:

$$CRA(\gamma) = -\frac{\gamma U''(\gamma)}{U'(\gamma)}.$$ 

- larger CRA $\Rightarrow$ “more concave” $U$. 

Theorem: If each user $n \in N$ has a positive minimum transmission power $P_{min}^n$, and $CRA(\gamma_n) \in [1, 2]$ for any values of $\gamma_n$, then there is a unique optimal solution of NUM Problem, and the ADP algorithm globally converges to it.

Proof: relating this algorithm to a fictitious supermodular game.

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**Theorem**: If each user $n \in N$

- has a positive minimum transmission power $P^\text{min}_n$, and
- $CRA(\gamma_n) \in [1, 2]$ for any values of $\gamma_n$,

then there is a unique **optimal** solution of **NUM** Problem, and the ADP algorithm **globally** converges to it.

**Proof**: relating this algorithm to a **fictitious supermodular game**.
Supermodular Games

- A class of games with **strategic complementaries**
  - Strategy sets are compact subsets of $\mathbb{R}$; and each player’s pay-off $S_n$ has increasing differences:
    \[
    \frac{\partial^2 S_n}{\partial x_n \partial x_m} > 0, \forall n, m.
    \]

- Key properties:
  - A PNE exists.
  - If the PNE is unique, then the **asynchronous** best response updates will globally converge to it.
Convergence Speed

- 10 users, log utilities.
- ADP algorithm (left figures) converges much faster than a gradient-based method (right figures).
Section 7.3: 4G Network Upgrade
When To Upgrade From 3G to 4G?

- **Early upgrade:**
  - More expensive, as cost decreases over time
  - Starts with few users, hence a small initial revenue

- **Late upgrade:**
  - Leads to a smaller market share
  - Delays 4G revenues

- **Need to**
  - Capture the above tradeoffs
  - Consider the *dynamics of users* adopting 4G and switching providers
  - Understand the *upgrade timing* between competing cellular providers
Duopoly Model

- Two competing operators
  - Initially both using 3G technology
  - Operator $i$ decides to upgrade to 4G at time $T_i$
  - Each operator wants to maximize its long-term profit

- What will be the equilibrium of $(T_1^*, T_2^*)$?
Users Switching

- W.L.O.G., assume $T_1 < T_2$
- Three time periods: $[0, T_1]$, $(T_1, T_2]$, and $(T_2, \infty)$
Users Switching

- W.L.O.G., assume $T_1 < T_2$
- Three time periods: $[0, T_1]$, $(T_1, T_2]$, and $(T_2, \infty)$
- When $t \in [0, T_1]$: No user switching.
Users Switching

- When \( t \in (T_1, T_2] \): both inter- and intra-operator user switching

Model: Customer migrations

Provider 1: 4G, 3G
Provider 2: 3G

\[ \lambda \] and \( \alpha \lambda \) transitions

- Customers switch providers to get 4G, at rate \( \lambda \).
- Customers of one provider upgrade to 4G at rate \( \alpha \lambda \).
Users Switching

- When $t \in (T_1, T_2]$: both inter- and intra- operator user switching

- When $t \in (T_2, \infty)$: only intra-operator user switching
Network Value (Revenue)

- Network value depends on the number of subscribers
  - Assume that operator $i$ has $N_i$ 4G users, $i = 1, 2$
  - Total 4G network value is $(N_1 + N_2) \log(N_1 + N_2)$ (network effect)
  - Operator $i$’s network value (revenue) is $N_i \log(N_1 + N_2)$

Later upgrade ⇒ take advantage of existing 4G population

The revenue for 3G network is similar, with an coefficient $\gamma \in (0, 1)$

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When should an operator upgrade from 3G to 4G?

Model:

Revenue

\[ R_i(t) = N_i(t) N_{i\uparrow}(t) + (N_i(t) - N_{i\downarrow}(t)) N_{i\uparrow}(t) \]

- \( N_i(t) \): number of users of provider \( i \)
- \( N_{i\uparrow}(t) \): number of 4G users of provider \( i \)
- \( N_{i\downarrow}(t) \): number of users of other provider (1 or 2)

4G calls cost 1, 3G calls cost 2,

\[ R_1(t) > R_2(t) \]

\[ \text{Profit} = \int_0^T e^{UT_i} R_i(t) \, dt \]

- \( U \): decrease rate of technology cost
- \( T \): discounting rate

Diagram:

- \( R_1(t) \)
- \( R_2(t) \)
- MS (market share) changing
- 4G network changes
- 1, 3G, 2, 3G, 2, 4G, 1, 4G networks
Upgrade Cost and Time Discount

- One-time upgrade cost:
  - $K$ at time $t = 0$
  - Discounted over time: $K \exp(-Ut)$

- Revenue is also discounted over time by $\exp(-St)$

- Earlier upgrade $\Rightarrow$ larger revenue and larger cost
Equilibrium Timings

Operator 1's equilibrium time $T_1^*$
Operator 2's equilibrium time $T_2^*$

NE 1: $T_1^* \leq T_2^*$
NE 2: $T_1^* \geq T_2^*$

Low cost regime: $0 = T_1^* = T_2^*$ as $K \uparrow$
Medium cost regime: $0 = T_1^* < T_2^* \uparrow$ as $K \uparrow$
High cost regime: $0 < T_1^* \uparrow < T_2^* \uparrow$ as $K \uparrow$

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Equilibrium Profits

Operator 1's equilibrium profit $\pi_1^*$

Operator 2's equilibrium profit $\pi_2^*$

Medium cost regime:
$\pi_1^* \uparrow < \pi_2^* \downarrow$ as $K \uparrow$

High cost regime:
$\pi_1^* \uparrow < \pi_2^* \uparrow$ as $K \uparrow$

Low cost regime:
$\pi_1^* \downarrow = \pi_2^* \downarrow$ as $K \uparrow$

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Section 7.4: Chapter Summary
Key Concepts

- **Theory**
  - Positive and negative Externality
  - Market failure
  - Pigovian tax
  - Network effect

- **Application**
  - Distributed wireless power control based on Pigovian tax
  - Cellular network upgrade considering network effect
References and Extended Reading


http://ncel.ie.cuhk.edu.hk/content/wireless-network-pricing