Wireless Network Pricing
Chapter 5: Monopoly and Price Discriminations

Jianwei Huang & Lin Gao

Network Communications and Economics Lab (NCEL)
Information Engineering Department
The Chinese University of Hong Kong
E-Book freely downloadable from NCEL website: http://ncel.ie.cuhk.edu.hk/content/wireless-network-pricing

Physical book available for purchase from Morgan & Claypool (http://goo.gl/JFG1ai) and Amazon (http://goo.gl/JQKaEq)
Chapter 5: Monopoly and Price Discriminations
Focus of This Chapter

- **Key Focus**: This chapter focuses on the problem of profit maximization in a monopoly market, where one service provider (monopolist) dominates the market and seeks to maximize its profit.

- **Theoretic Approach**: Price Theory
  - Price theory mainly refers to the study of how prices are decided and how they go up and down because of economic forces such as changes in supply and demand (from Cambridge Business English Dictionary).
Follow the discussions in “Price Theory and Applications” by B. Peter Pashigian (1995) and Steven E. Landsburg (2010)

- **Part I: Monopoly Pricing**
  - The service provider charges a single optimized price to all the consumers;

- **Part II: Price Discrimination**
  - The service provider charges different prices for different units of products or to different consumers.
Section 5.1
Theory: Monopoly Pricing
What is Monopoly?

- Etymology suggests that a “monopoly” is a single seller, i.e., the only firm in its industry.
  - Question: Is Apple a monopoly?
    - It is the only firm that sells iPhone;
    - It is not the only firm that sells smartphones.

- The formal definition of monopoly is based on the monopoly power.

**Definition (Monopoly)**

A firm with monopoly power is referred to as a monopoly or monopolist.
What is Monopoly Power?

- Monopoly power (or market power) is the ability of a firm to affect market prices through its actions.

**Definition (Monopoly Power)**

A firm has monopoly power, if and only if

(i) it faces a downward-sloping demand curve for its product, and

(ii) it has no supply curve.

- (i) implies that a monopolist is not perfectly competitive. That is, he is able to set the market price so as to shape the demand.
- (ii) implies that the market price is a consequence of the monopolist’s actions, rather than a condition that he must react to.
**Profit Maximization Problem**

- $P$: the market price that a monopolist chooses;
- $Q \triangleq D(P)$: the downward-sloping demand curve that the monopolist faces;

**Definition (Monopolist’s Profit Maximization Problem)**

How should the monopolist choose the market price $P$ to maximize his profit $\pi(P)$, where

$$\pi(P) \triangleq P \cdot Q = P \cdot D(P).$$
Profit Maximization Problem

- The **first-order** condition:
  \[
  \frac{d\pi(P)}{dP} = Q + P \cdot \frac{dQ}{dP} = 0
  \]

- The **optimality** condition:
  \[
  \frac{P \cdot \Delta Q}{Q \cdot \Delta P} + 1 = 0
  \]

  ▶ \( \Delta P \) is a **very small** change in price, and \( \Delta Q \) is the corresponding change in demand quantity.
Demand Elasticity

- **Price Elasticity of Demand** (defined in Section 3.2.5)

\[
\eta \triangleq \frac{\Delta Q/Q}{\Delta P/P} = \frac{P \cdot \Delta Q}{Q \cdot \Delta P}
\]

- The ratio between the percentage change of demand and the percentage change of price.

- A Closely Related Question: *Under a particular price* \(P\) *and demand* \(Q = D(P)\), *how much does the monopolist have to lower his price to sell additional* \(\Delta Q\) *units of product?*

  \[
  \Delta P = \frac{P}{Q \cdot \eta} \cdot \Delta Q
  \]
Demand Elasticity

- The monopolist’s total profit changes by selling additional $\triangle Q$ units of product:

$$
\triangle \pi \triangleq P \cdot \triangle Q - |\triangle P| \cdot Q
$$

$$
= P \cdot \triangle Q - \left| \frac{P}{Q \cdot \eta} \cdot \triangle Q \right| \cdot Q
$$

$$
= P \cdot \triangle Q \cdot \left( 1 - \frac{1}{|\eta|} \right)
$$

- $P \cdot \triangle Q$ is the profit gain that the monopolist achieves, by selling additional $\triangle Q$ units of product at price $P$;
- $|\triangle P| \cdot Q$ is the profit loss that the monopolist suffers, due to the decrease of price (by $|\triangle P|$) for the previous $Q$ units of product.
Demand Elasticity

- Monopolist’s Total Profit Change:

\[ \Delta \pi = P \cdot \Delta Q \cdot \left(1 - \frac{1}{|\eta|}\right) \]

- If \( |\eta| > 1 \), then \( \Delta \pi > 0 \). This implies that the monopolist has incentive to decrease the price when \( |\eta| > 1 \).
- If \( |\eta| < 1 \), then \( \Delta \pi < 0 \). This implies that the monopolist has incentive to increase the price when \( |\eta| < 1 \).
- If \( |\eta| = 1 \), then \( \Delta \pi = 0 \). This implies that the monopolist has no incentive to increase or decrease the price when \( |\eta| = 1 \) (assuming no producing cost).

- The price under \( |\eta| = 1 \) is the optimal price (if no producing cost)
  - Equivalent to the previous first-order condition.
Demand Elasticity

- Suppose the unit producing cost is $C$. Then, the **optimal price** is given by $\Delta \pi = \Delta C \triangleq C \cdot \Delta Q$, or equivalently,

$$P \cdot \Delta Q \cdot \left(1 - \frac{1}{|\eta|}\right) = C \cdot \Delta Q.$$  

- Hence at the optimal price, we have

$$|\eta| = \frac{1}{1 - C/P} > 1.$$  

- Recall that
  - **When** $|\eta| > 1$, we say that the demand curve is **elastic**.
  - **When** $|\eta| < 1$, we say that the demand curve is **inelastic**.

**Theorem**

*A monopolist always operates on the elastic portion of the demand curve.*
Demand Elasticity

- When $\triangle Q = 1$, then
  - $\triangle \pi = P \cdot \left(1 - \frac{1}{|\eta|}\right)$ is usually called the marginal revenue (MR);
  - $\triangle C$ is usually called the marginal cost (MC);

- Hence the optimal monopoly price **equalizes** the marginal revenue and marginal cost,

$$\triangle \pi = \triangle C.$$
Section 5.2
Theory: Price Discriminations
What is Price Discrimination?

- **Price discrimination** (or price differentiation) is a pricing strategy where products are transacted at **different prices** in different markets or territories.

- **Examples of Price Discriminations:**
  - Charge different prices to the same consumer, e.g., for different units of products;
  - Charge uniform but different prices to different groups of consumers for the same product.

- **Types of Price Discrimination**
  - **First-degree** price discrimination
  - **Second-degree** price discrimination
  - **Third-degree** price discrimination
An Illustrative Example

Example: *How the monopolist increase his profit via price discrimination?*

- **MR**: the marginal revenue (profit) curve;
- **MC**: the marginal cost curve;
- **Demand**: the downward-sloping demand curve;
Without Price Discrimination

- Without price discrimination, the monopolist charges a single monopoly price to all consumers:

- The optimal monopoly price is $P^*$ and the demand is $Q^*$ (intersection of MC and MR curves);

- The monopolist’s profit is $\pi^*$, and the consumer surplus is $\pi^+$. 
With price discrimination, the monopolist can charge different prices to different consumers:

For example, the monopolist can charge each consumer the most that he would be willing to pay for each product that he buys;

With the same demand $Q^*$, the monopolist’s profit is $\pi^* + \pi^+$, and the consumer surplus is 0;

When the demand increases to $Q^*$, the monopolist’s profit is $\pi^* + \pi^+ + \pi^*$, and the consumer surplus is 0;
First Degree Price Discrimination

- With the **first-degree price discrimination** (or perfect price discrimination), the monopolist charges each consumer the most that he would be willing to pay for each product that he buys.

- The monopolist captures all the market surplus, and the consumer gets zero surplus.

- It requires that the monopolist knows exactly the maximum price that every consumer is willing to pay for each product, i.e., the full knowledge about every consumer demand curve.
Illustration of First Degree Price Discrimination

- The consumer is willing to pay a maximum price $P_1$ for the first product, $P_2$ for the second product, and so on.
- Under the first-degree price discrimination, the consumer is charged by $P_1$ for the first product, $P_2$ for the second product, and so on.
- The monopolist captures all the market surplus (shadow area).
Second Degree Price Discrimination

- With the **second-degree price discrimination** (or declining block pricing), the monopolist offers a **bundle of prices** to each consumer, with different prices for different blocks of units.

- The second-degree price discrimination can be viewed as a **limited version** of the first-degree price discrimination (where a different price is set for every different unit).

- The second-degree price discrimination can be viewed as a **generalized version** of the monopoly pricing (as it degrades to the monopoly pricing when the number of prices is one).
Illustration of Second Degree Price Discrimination

- Under this second-degree price discrimination, the monopolist offers a bundle of prices \( \{P_1, P^*, P_2\} \) with \( P_1 > P^* > P_2 \).
  - \( P_1 \) is the unit price for the first block (the first \( Q_1 \) units) of products;
  - \( P^* \) is the unit price for the second block (from \( Q_1 \) to \( Q^* \)) of products;
  - \( P_2 \) is the unit price for the third block (from \( Q^* \) to \( Q_2 \)).
  - The monopolist’s profit is illustrated by the shadow area, and the consumer surplus is \( \delta_1 + \delta^* + \delta_2 \).
Illustration of Second Degree Price Discrimination

- Under the second-degree price discrimination \( \{P_1, P^*, P_2\} \):
  - The monopolist’s profit is illustrated by the shadow area, and the consumer surplus is \( \delta_1 + \delta^* + \delta_2 \).

- Under the first-degree price discrimination:
  - The monopolist charges a different price \( D(Q) \) for each unit of product;
  - The monopolist captures all the market surplus (the shadow area + \( \delta_1 + \delta^* + \delta_2 \), and the consumer achieves a zero surplus.)
Illustration of Second Degree Price Discrimination

- Under the second-degree price discrimination \( \{P_1, P^*, P_2\} \):
  - The monopolist’s profit is illustrated by the shadow area, and the consumer surplus is \( \delta_1 + \delta^* + \delta_2 \).

- Under the monopoly pricing (without price discrimination):
  - The optimal monopoly price is \( P^* \) and the demand is \( Q^* \);
  - The monopolist’s profit is \( P^* \cdot Q^* \), and the consumer surplus is \( \delta_1 + \delta^* + (P_1 - P^*) \cdot Q_1 \).

![Diagram of price and quantity with profit and consumer surplus areas labeled.](image)
Second Degree Price Discrimination

- **Comparison** of Different Pricing Strategies
  - When the number of prices is one, the second-degree price discrimination degrades to *the monopoly pricing*;
  - When the price bundle curve approximates to the inverse demand curve $P(Q)$, the second-degree price discrimination converges to *the first-degree price discrimination*. 
Third Degree Price Discrimination

- **Limitation** of First- and Second-Degree Price Discriminations
  - Needs the full or partial demand curve information of every individual consumer, and benefits from this information by charging the consumer different prices for different units of products.

- A Natural Question: *Whether (and how, if so) the monopolist discriminates the price, if he does not know the detailed demand curve information of each individual consumer, but knows from experience that different groups of consumers have different total demand curves?*

  → **Third-Degree Price Discrimination**
**Third Degree Price Discrimination**

- With the **third-degree price discrimination** (or multi-market price discrimination), the monopolist specifies different prices for different consumer groups (with different total demand curves).
  - Example: The Disney Park offers different ticket prices to three player groups: children, adults, and elders.

- Third-degree price discrimination usually occurs when
  - the monopolist faces multiple identifiable different groups of consumers with different total demand curves;
  - the monopolist knows the total demand curve of every consumer group (but not the individual demand curve of each consumer).
How to Identify Customers?
By Age

Don't forget my...

SENIOR DISCOUNT
By Time

- **Kindle 2**
  - 02/2009: $399
  - 07/2009: $299
  - 10/2009: $259
  - 06/2010: $189
Dynamic Pricing

Retailers are adjusting prices on everyday items several times a day. Here is a look at prices for a GE microwave on Aug. 12 at three Web retailers:

- **SEARS**
  - $900
  - 5:31 p.m. Best Buy raises price to $899.99

- **AMAZON**
  - $900
  - 6:25 p.m. Amazon drops price to $744.46
  - 7:43 p.m. Best Buy returns price to $809.99
  - 8:33 p.m. Amazon raises price to $856.08

- **BEST BUY**
  - $850
  - 6:25 p.m. Amazon drops price to $744.46
  - 7:43 p.m. Best Buy returns price to $809.99

Source: Decide.com
Graphic by Alberto Cervantes/The Wall Street Journal
More Innovative Ways
Consider a simple scenario:

- Two groups (markets) of consumers:
  - The total demand curve in each market \( i \in \{1, 2\} \) is \( D_i(P) \);
  - The monopolist decides the price \( P_i \) for each market \( i \).

Key problem: How should the monopolist set the prices \( \{P_1, P_2\} \) to maximize his profit?

- Whether to charge the same price or different prices in different markets (groups)?
- Which market should get the lower price if the monopolist charges different prices?
- What is the relationship between the prices of two markets?
Third Degree Price Discrimination

- The monopolist’s profit \( \pi(P_1, P_2) \) under prices \( \{P_1, P_2\} \) is

\[
\pi(P_1, P_2) \triangleq P_1 \cdot Q_1 + P_2 \cdot Q_2 - C(Q_1 + Q_2)
\]

- The first-order condition:

\[
\frac{\partial \pi(P_1, P_2)}{\partial P_i} = Q_i + P_i \cdot \frac{dQ_i}{dP_i} - C'(Q_1 + Q_2) \cdot \frac{dQ_i}{dP_i} = 0
\]

- \( Q_i \triangleq D_i(P_i) \) is the demand curve in market \( i \);
- \( \eta_i \triangleq \frac{P_i}{Q_i} \frac{dQ_i}{dP_i} \) is the price elasticity of demand in market \( i \);
- \( C'(Q_1 + Q_2) \) is the marginal cost (MC) of the monopolist;
The optimality condition:

\[ C'(Q_1 + Q_2) = P_i + Q_i \cdot \frac{dP_i}{dQ_i} = P_i \cdot \left(1 - \frac{1}{|\eta_i|}\right) \]

⇒ Under the optimal prices \((P_1^*, P_2^*)\), the marginal revenues (MR) in all markets are identical, and are equal to the marginal cost (MC):

\[ P_1^* \cdot \left(1 - \frac{1}{|\eta_1|}\right) = P_2^* \cdot \left(1 - \frac{1}{|\eta_2|}\right) \]

- \(P_i \cdot \left(1 - \frac{1}{|\eta_i|}\right)\) is the marginal revenue (MR) of the monopolist in market \(i\);
Third Degree Price Discrimination

The optimal prices \((P^*_1, P^*_2)\) satisfy

\[
P^*_1 \cdot \left(1 - \frac{1}{|\eta_1|}\right) = P^*_2 \cdot \left(1 - \frac{1}{|\eta_2|}\right)
\]

- If \(|\eta_1| \neq |\eta_2|\), then \(P^*_1 \neq P^*_2\). That is, the monopolist will charge different prices when two markets have different price elasticities.
- If \(|\eta_1| > |\eta_2|\), then \(P^*_1 < P^*_2\). That is, the market with the higher price elasticity will get a lower optimal price.
Third Degree Price Discrimination

- Graphic Interpretation of Optimal Prices \((P_1^*, P_2^*)\)
  - \(D_i\): the demand curve in market \(i\);
  - \(MR_i\): the marginal revenue curve in market \(i\);
  - \(MR\) (the blue curve): the overall marginal revenue curve (summing \(MR_1\) and \(MR_2\) horizontally);
  - \(MC\) (the red curve): the marginal cost curve;

![Graphic Interpretation of Optimal Prices](image)
Third Degree Price Discrimination

- Graphic Interpretation of Optimal Prices \((P^*_1, P^*_2)\)
  - Market 1: the demand is \(Q_1\), the marginal revenue equals \(C_0\);
  - Market 2: the demand is \(Q_2\), the marginal revenue equals \(C_0\);
  - Total market demand is \(Q_1 + Q_2\), and the marginal cost is \(C_0\);
  - \(C_0\) is at the intersection of MC and MR curves.

![Graphical representation of third degree price discrimination](image)

- Price
- Quantity
- \(P^*_1\), \(P^*_2\), \(C_0\), \(Q_1\), \(Q_2\), \(Q_1 + Q_2\)
- MC, MR
Third Degree Price Discrimination

* Necessary conditions to make the third-degree price discrimination applicable and profitable:
  
  ▶ **Monopoly power**: The firm must have the monopoly power to affect market price (there is no price discrimination in perfectly competitive markets).
  
  ▶ **Market segmentation**: The firm must be able to split the market into different groups of consumers, and also be able to identify the type of each consumer.
  
  ▶ **Elasticity of demand**: The price elasticities of demand in different markets are different.