

S-MAPEL: Monotonic Optimization for Non-convex Joint Power Control and Scheduling Problems

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Abstract—In interference-limited wireless networks where simultaneous transmissions on nearby links heavily interfere with each other, power control alone is not sufficient to eliminate strong levels of interference between close-by links. In this case, scheduling, which allows close-by links to take turns to be active, plays a crucial role for achieving high system performance. Joint power control and scheduling that maximizes the system utility has long been a challenging problem. The complicated coupling between the signal-to-interference ratio of concurrently active links as well as the flexibility to vary power allocation over time gives rise to a series of non-convex optimization problems, for which the global optimal solution is hard to obtain. This paper is a first attempt to solve the non-convex joint power control and scheduling problems efficiently in a global optimal manner. In particular, it is the monotonicity rather than convexity of the problem that we exploit to devise an efficient algorithm, referred to as S-MAPEL, to obtain the global optimal solution. To further reduce the complexity, we propose an accelerated algorithm, referred to as A-S-MAPEL, based on the inherent symmetry of the optimal solution. The optimal joint-power-control-and-scheduling solution obtained by the proposed algorithms serves as a useful benchmark for evaluating other existing schemes. With the help of this benchmark, we find that on-off scheduling is of much practical value in terms of system utility maximization if "off-the-shelf" wireless devices are to be used.

Index Terms—Power Control, Scheduling, Non-convex Optimization, Global Optimization, Monotonic Optimization, Decomposition.

I. INTRODUCTION

A. Motivation and Related Work

Driven by the increasing popularity of wireless broadband services, future wireless networks are expected to provide high-data-rate services to densely populated user environments. Due to the broadcast nature of wireless medium, simultaneous transmissions on nearby wireless links interfere with each other, thus adversely affecting data rates and Quality of Service (QoS) in the system. Mitigating interference is therefore a fundamental issue that must be addressed in next generation wireless networks.

Transmission power control in wireless networks has been extensively studied over the last two decades as an important mechanism to mitigate the adverse effect of interference (see a recent survey [1]). Thanks to the explosive growth of wireless

data services, there has been tremendous recent interest in finding an optimal transmission power allocation that maximizes a system-wide efficiency metric (i.e., utility) while satisfying the individual user's QoS requirements [2]–[7]. However, due to the complicated interference coupling between links, optimal power control is known to be a non-convex optimization problem, and hence is difficult to solve despite its paramount importance. A majority of efforts in solving the power control problem aim to convexify it through transformation [4], reparameterization [7], relaxation [5], and approximation [4], oftentimes compromising the global optimality. As a result, achieving the global optimal power allocation had been a long-standing open problem until our very recent work in [2] that proposed the MAPEL algorithm to solve the problem using the latest development of monotonic optimization (MO) [12].

In dense networks, power control alone is not sufficient to eliminate strong levels of interference between close-by links. In this case, scheduling, which allows close-by links to take turns to be active, is indispensable. This is illustrated in the following motivating example.

Motivating Example: Consider a two-link network as shown in Fig. 1, where links are close to each other. Assume that the maximum allowable transmission power of each link is 1.0W. Meanwhile, the noise power at each receiver is 10^{-4} W. In this case, the achievable data rate region by using power control only is given Fig. 2(a), where each point in the data rate region corresponds to a feasible power allocation strategy. On the other hand, if we integrate power control with scheduling, the achievable data rate region is considerably enlarged and becomes convex, as shown in Fig. 2(b). For further illustration, let us take a specific utility, namely proportional fair utility $U(r_1, r_2) = \log(r_1) + \log(r_2)$, as an example, where r_1 and r_2 denote the data rates of link 1 and link 2, respectively. The optimal power-control and joint-power-control-and-scheduling strategies that maximize the utility function are shown in Table I. It can be seen that without scheduling, the data rates as well as the system utility are low due to severe co-channel interference. In contrast, the joint-power-control-and-scheduling strategy leads to much higher utility and data rates. This motivating example suggests that it can be spectrally inefficient to operate with power control alone in dense wireless networks. Joint power control and scheduling is a natural remedy to alleviate interference in this case.

Joint power control and scheduling is by nature much more challenging than pure power control, for the problem involves not only complicated coupling between mutual interference of concurrently active links, but also that between the power allocation during different time periods. In [8] and [9], the authors

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TABLE I
COMPARISON OF PURE POWER CONTROL VERSUS JOINT POWER CONTROL AND SCHEDULING

	Power control without scheduling		Joint power control and scheduling	
	Optimal transmission power		Optimal transmission power and scheduling	
	$p_1^* = 1.0\text{W}$	$p_2^* = 0.71\text{W}$	$p_1^*(t) = 1.0\text{W}, \forall t \in [0, 0.5T]$ $p_1^*(t) = 0\text{W}, \forall t \in [0.5T, T]$	$p_2^*(t) = 0\text{W}, \forall t \in [0, 0.5T]$ $p_2^*(t) = 1.0\text{W}, \forall t \in [0.5T, T]$
Data rate	$r_1^* = 1.9294\text{bps/Hz}$	$r_2^* = 1.9390\text{bps/Hz}$	$r_1^* = 4.9836\text{bps/Hz}$	$r_2^* = 5.4833\text{bps/Hz}$
Overall utility	1.3194		3.3079	

* T is a given period of time, in which the channel gains keep constant.

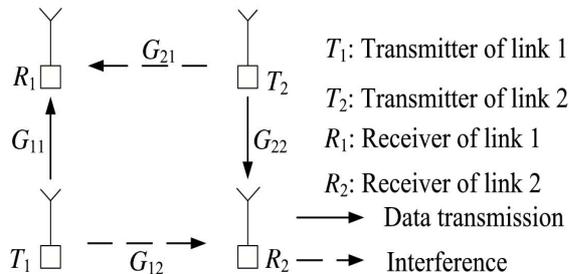


Fig. 1. A two-link network with $G_{11} = 0.1$, $G_{22} = 0.2$, $G_{12} = G_{21} = 0.05$, where G_{ij} denotes the channel gain from node T_i and node R_j .

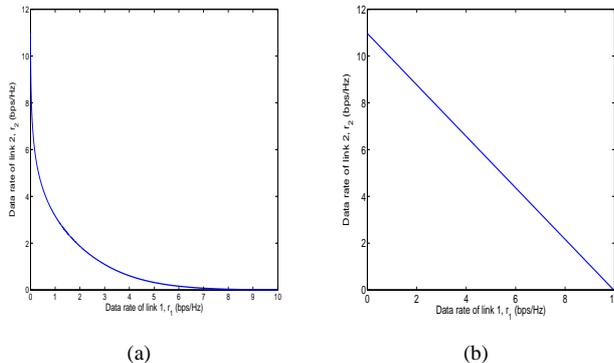


Fig. 2. The data rate regions obtained by (a) pure power control strategy and (b) joint power control and scheduling strategy, respectively.

aimed at scheduling as many link transmissions as possible in a given period of time, subject to the constraint on the minimum SINR requirement of each link. Utility-maximizing power controlled scheduling was previously studied in [10] and [11] under different contexts. Ref. [10] studied a downlink code division multiple access (CDMA) system, where a dual problem was solved to obtain the solution. Due to the non-convexity of the problem, however, the duality gap is non-zero. Hence, the solution obtained in [10] is not guaranteed to be optimal. Cruz et al investigated joint routing, scheduling and power control in multihop wireless networks in [11]. This paper focuses on a low-SINR regime, so that the data rate is a linear function of SINR. In this case, the optimal allocation is simply a linear combination of extremal points, and hence easy to obtain. However, the low-SINR assumption may not be valid in practice, as links tend to maintain a reasonably high SINR for signal reception quality. Notably, optimal joint power control and scheduling that maximizes system utility in a general SINR region has remained an open problem to this day, despite the tremendous efforts that have been devoted to

this subject.

In this paper, we address this problem by proposing an efficient algorithm, referred to as S-MAPEL (where the prefix ‘‘S’’ stands for Scheduling), to obtain the global optimal solution of the joint power control and scheduling problem. The proposed algorithm complements our previous work MAPEL [2] that has only considered power control without scheduling. One key distinction of the problem formulation in this paper is that the system utility function is allowed to be *either concave or non-concave*, as long as it is monotonic. The results of this work address the following important questions: (i) given a wireless network, which links should transmit together and which should not; (ii) how much power and for how long should each link transmit to maximize the overall system utility while satisfying individual user’s QoS requirements.

B. Contributions

The main contributions of this paper are summarized as follows:

- We formulate the non-convex joint power control and scheduling problem into a MO problem that is amenable to an efficient global optimization algorithm. There are three key observations that lead to such a formulation. First, by a standard convexity argument, the joint power control and scheduling problem is equivalent to seeking a piecewise constant power allocation that has $M + 1$ (M : the number of links) degrees of freedom in the time domain. Second, the objective function of the optimization problem is monotonically increasing in both SINR and scheduling period. Last, the feasible set of the equivalent transformed problem, although may be not convex, is always normal¹. These properties allow us to bypass the non-convexity of the problem and exploit monotonicity to devise efficient solution algorithms.
- We propose an algorithm, referred to as S-MAPEL, to obtain the global optimal solution of the joint power control and scheduling problem. The MAPEL algorithm proposed in our previous work [2] for solving pure power control problems cannot be directly applied to the joint power control and scheduling problem due to convergence issues that will be discussed later. Being a non-trivial extension of MAPEL, S-MAPEL guarantees to converge to the global optimal solution even for non-concave system utilities. By tuning a small error

¹Various math preliminaries and definitions are given in Sections IV and V.

tolerance, we can engineer a flexible tradeoff between system performance and convergence time.

- To further reduce the computational complexity, an accelerated algorithm, referred to as A-S-MAPEL, is proposed based on the inherent symmetry of the optimal solutions. Our results show that A-S-MAPEL greatly reduces the convergence time, while yielding a negligible performance degradation compared with S-MAPEL.
- We use the optimal and near optimal solutions obtained by the proposed algorithms as a benchmark to evaluate the performance of existing heuristics. An interesting conclusion is that on-off power control achieves close-to-optimal performance when it is integrated with scheduling. On the other hand, without scheduling, on-off power control can be far from optimal. This implies that scheduling is an indispensable component in wireless system design, if off-the-shelf wireless devices that do not offer the freedom to adjust transmission power are to be used.

C. Organization

The rest of this paper is organized as follows. Section II introduces the system model and the problem formulation. In Section III, we reformulate the joint power control and scheduling problem into a MO problem with finite size. Some properties of the feasible region of MO problem are also discussed. The S-MAPEL algorithm is proposed and analyzed in Section IV. Section V presents an accelerated algorithm, referred to as A-S-MAPEL, to reduce the computational complexity for the joint power control and scheduling problem. In Section VI, we evaluate the performance of the proposed algorithms through several simulations. The paper is concluded in Section VII.

II. SYSTEM FORMULATION

We consider a single-hop wireless ad hoc network with a set $\mathcal{M} = \{1, \dots, M\}$ of *distinct* links. Each link consists of a transmitter node T_i and a receiver node R_i . The channel gain between node T_i and node R_j is denoted by G_{ij} , which is determined by various factors such as path loss, shadowing and fading effects. To condense notation, we write the channel gains into a channel matrix form $\mathbf{G} = [G_{ij}]$. Assume that the channel gains are constant during the time interval T under consideration. For such a network, there are typically two strategies in the current literature, to maximize the overall system utility, namely *pure power control* and *joint power control and scheduling*. In what follows, we show how these two strategies can be formulated into optimization problems that are generally non-convex. Notably, the joint power control and scheduling problem is much more complicated due to the additional freedom in the time domain, although pure power control is already a NP-hard problem by itself.

A. Preliminary Work—Power Control without Scheduling

Pure power control schemes without scheduling are in general the ones that allocate constant power levels to links for

the entire period of interest T . Let p_i denote the transmission power of link i (i.e., from node T_i), with P_i^{\max} being its maximum allowable value. For notational convenience, we write $\mathbf{p} = (p_i, \forall i \in \mathcal{M})$ and $\mathbf{P}^{\max} = (P_i^{\max}, \forall i \in \mathcal{M})$ as the transmission power vector and the maximum transmission power vector, respectively. Likewise, let the receiving noise on link i be n_i . Thus, the received SINR of link i is

$$\gamma_i(\mathbf{p}) = \frac{G_{ii}p_i}{\sum_{j \neq i} G_{ji}p_j + n_i}, \quad (1)$$

and the corresponding data rate r_i calculated based on the Shannon capacity formula is $\log_2(1 + \frac{\gamma_i(\mathbf{p})}{\Gamma})$, where Γ is the SINR gap that reflects a particular modulation and coding scheme. Without loss of generality, we assume $\Gamma = 1$ hereafter.

We aim to find the optimal power allocation \mathbf{p}^* that maximizes the overall system utility subject to individual user's QoS requirements. To satisfy the QoS requirement, each link must maintain a minimum data rate r_i^{\min} . Mathematically, the optimal power control is formulated into the following form²:

$$\begin{aligned} & \underset{\mathbf{p}}{\text{maximize}} && U(\mathbf{r}) \\ & \text{subject to} && r_i = \log_2(1 + \gamma_i(\mathbf{p})) \geq r_i^{\min}, \forall i \in \mathcal{M} \quad (P_p) \\ & && \mathbf{0} \preceq \mathbf{p} \preceq \mathbf{P}^{\max}, \end{aligned}$$

where $U(\mathbf{r})$ represents the system-wide utility, with \mathbf{r} being the vector of r_i . It is often assumed to be additive across links, i.e., $U(\mathbf{r}) = \sum_{i=1}^M U_i(r_i)$, with $U_i(r_i)$ being the utility of link i . Typically, $U_i(r_i)$ is either a concave non-decreasing function for elastic traffic or a non-concave non-decreasing function for delay-sensitive traffic. By appropriately choosing the utility function $U_i(r_i)$, we can strike different balances between spectrum efficiency and fairness. For elastic traffics, a general form of the utility function $U_i(r_i)$ is the generalized α -fairness utility function, i.e.,

$$U_i(r_i) = \begin{cases} \log(r_i) & \text{if } \alpha = 1, \\ \frac{(r_i)^{1-\alpha}}{1-\alpha} & \text{if } \alpha \geq 0 \text{ and } \alpha \neq 1. \end{cases} \quad (2)$$

Maximizing the objective function as given in (2) corresponds to maximizing the average total throughput when $\alpha = 0$, the proportional fairness when $\alpha = 1$, and the max-min fairness when $\alpha \rightarrow \infty$. This implies that increasing α leads to fairer allocation. For delay-sensitive traffics, sigmoidal utility functions $U_i(r_i)$ as given in Fig. 3 capture the ‘‘happiness’’ of links. In general, such utility function in the standard form is expressed as:

$$U_i(r_i) = \frac{1}{1 + \exp(-a_i(r_i - b_i))}, \quad (3)$$

where $\{a_i, b_i\}$ are constant positive integers. From Fig. 3, it can be seen that b_i can be regarded as a kind of threshold such that $U_i(r_i)$ is concave when r_i is beyond b_i , and convex otherwise. On the other hand, a_i can be regarded as an

²In this paper, $\mathbf{0}$ is a vector with every element being 0, and $\mathbf{1}$ is a vector with every element being 1. The notation ‘‘ \preceq ’’ represents the component-wise smaller than or equal relation, and the notation ‘‘ \succeq ’’ represents component-wise larger than or equal relation.

indication of slope such that the larger a_i , the steeper the curve of $U_i(r_i)$.

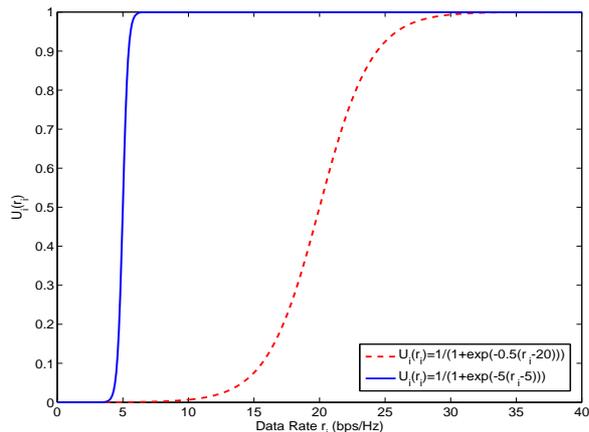


Fig. 3. An example of sigmoidal function.

Due to the complicated coupling of γ_i across links, Problem P_p is in general non-convex even if $U_i(r_i)$ is a concave function, let alone the cases with non-concave $U_i(r_i)$'s. Consequently, obtaining a global optimal power allocation had remained an open problem until our very recent work in [2] where MAPEL is proposed to solve such a problem using the techniques developed in the literature of monotonic optimization.

B. Joint Power Control and Scheduling

The motivating example in Introduction reveals that power control alone is not sufficient to handle interference when links are close to each other. In dense networks, joint power control and scheduling leads to much higher system utility and throughput than pure power control schemes. Specifically, the joint power control and scheduling problem can be formulated as follows

$$\begin{aligned} & \text{maximize}_{\{\mathbf{p}(t)\}} U(\mathbf{r}) \\ & \text{subject to } r_i = \frac{1}{T} \int_0^T \log_2(1 + \gamma_i(\mathbf{p}(t))) dt \geq r_i^{\min}, \forall i \in \mathcal{M} \\ & \mathbf{0} \preceq \mathbf{p}(t) \preceq \mathbf{P}^{\max}, \forall t \in [0, T], \end{aligned} \quad (P_c)$$

where scheduling has been integrated into the time-varying power allocation $\mathbf{p}(t)$, as choosing a subset of links to transmit during a certain time interval is equivalent to allocating zero transmit power to the unscheduled links and positive power to the scheduled ones.

Joint power control and scheduling in P_c is much more challenging than pure power control in P_p because of the following two reasons. First, the problem size of P_c is infinite as t is continuous during $[0, T]$. Second, the shape of the feasible set is further complicated by the coupling between $\mathbf{p}(t)$ at different t . Fortunately, a close look at P_c reveals that it possesses similar hidden monotonicity as the power control problem P_p , due to the fact that the problem maximizes an objective function monotonically increasing with SINR.

This opens the possibility to solve the problem through MO methods.

III. JOINT POWER CONTROL AND SCHEDULING AS MONOTONIC OPTIMIZATION

Before we can formulate Problem P_c into a MO problem that is amenable to efficient solutions, we must address the two challenges mentioned in the last sections, namely, infinite problem size and complicated shape of the feasible region. In the next subsection, we show how Problem P_c can be reformulated to get rid of the hurdles.

A. Discretization

1) The issue of infinite problem size

The issue of infinite problem size can be addressed by the following Lemma and Theorem. Before presenting the Lemma and Theorem, we first define achievable instantaneous data rate set $\mathcal{R}(t)$ and the achievable average data rate set $\bar{\mathcal{R}}$:

$$\begin{aligned} \mathcal{R}(t) = \left\{ \mathbf{r}(t) \mid r_i(t) = \log_2(1 + \gamma_i(\mathbf{p}(t))) \right. \\ \left. \text{and } \mathbf{0} \preceq \mathbf{p}(t) \preceq \mathbf{P}^{\max}, \forall i \in \mathcal{M} \right\}, \end{aligned} \quad (4)$$

and

$$\begin{aligned} \bar{\mathcal{R}} = \left\{ \mathbf{r} \mid r_i = \frac{1}{T} \int_0^T \log_2(1 + \gamma_i(\mathbf{p}(t))) dt \right. \\ \left. \text{and } \mathbf{0} \preceq \mathbf{p}(t) \preceq \mathbf{P}^{\max}, \forall i \in \mathcal{M}, \forall t \in [0, T] \right\}. \end{aligned} \quad (5)$$

The instantaneous data rate set $\mathcal{R}(t)$ is the set of all data rates achievable by power allocation at time instant t . On the other hand, the average data rate set $\bar{\mathcal{R}}$ is the set of all achievable average data rates during a scheduling period T through time-varying power allocation.

Remark 1: In this paper, $\mathcal{R}(t)$ is the same for all $t \in [0, T]$, since the channel is constant during the period of T .

By the standard convexity argument, Remark 1 leads to Lemma 1 [13].

Lemma 1: The achievable average data rate set $\bar{\mathcal{R}}$ is the convex hull of the instantaneous data rate set $\mathcal{R}(t)$, i.e., $\bar{\mathcal{R}} = \text{Convex Hull}\{\mathcal{R}(t)\}$.

Theorem 1: By Caratheodory theorem [13] and Lemma 1, the number of elements in $\bar{\mathcal{R}}(t)$ that is needed to construct an arbitrary average data rate vector $\mathbf{r} = (r_i, \forall i \in \mathcal{M})$ in $\bar{\mathcal{R}}$ is no more than $M + 1$.

Theorem 1 implies that an arbitrary average data rate vector \mathbf{r} can be achieved by dividing $[0, T]$ into $M + 1$ intervals with lengths $\beta^1, \dots, \beta^{M+1}$ and assigning power vectors $\mathbf{p}^1, \dots, \mathbf{p}^{M+1}$ to these intervals. In particular, if less than $M + 1$ intervals are needed for the optimal solution, then some β^k 's and \mathbf{p}^k 's are equal to zero. Therefore, the

constraints of Problem P_c can be replaced by

$$\begin{aligned} r_i &= \sum_{k=1}^{M+1} \beta^k \log_2(1 + \gamma_i(\mathbf{p}^k)) \geq r_i^{\min}, \forall i \in \mathcal{M} \\ \sum_{k=1}^{M+1} \beta^k &= 1, \boldsymbol{\beta} = (\beta^1, \dots, \beta^{M+1}) \succeq \mathbf{0} \\ \mathbf{0} &\preceq \mathbf{p}^k \preceq \mathbf{P}^{\max}, \forall k \in \mathcal{K} = \{1, 2, \dots, M+1\}, \end{aligned} \quad (6)$$

where we have normalized β^k with respect to T . By doing so, we have turned an infinite number of variables $\mathbf{p}(t)$ to a finite number of variables \mathbf{p}^k without compromising the optimality of the problem. The joint power control and scheduling optimization problem is now equivalent to find a piecewise constant power allocation that has $M+1$ degrees of freedom in the time domain.

2) The issue of complicated feasible region

Having transformed the problem into one with a finite number of variables, we are still confronted with the problem that the feasible region specified by (6) has a complicated shape. We address this problem by defining $\hat{U}_i(r_i)$ to be

$$\hat{U}_i(r_i) = \begin{cases} U_i(r_i) & \text{if } r_i \geq r_i^{\min}, \\ -\infty & \text{otherwise.} \end{cases} \quad (7)$$

Similar to $U_i(r_i)$, $\hat{U}_i(r_i)$ is a monotonic increasing function. With (7), Problem P_c can be rewritten as

$$\begin{aligned} \text{maximize}_{\boldsymbol{\beta}, \{\mathbf{p}^k, \forall k\}} & \sum_{i=1}^M \hat{U}_i\left(\sum_{k=1}^{M+1} \beta^k \log_2(1 + \gamma_i(\mathbf{p}^k))\right) \\ \text{subject to} & \sum_{k=1}^{M+1} \beta^k \leq 1, \boldsymbol{\beta} \succeq \mathbf{0} \\ & \mathbf{0} \preceq \mathbf{p}^k \preceq \mathbf{P}^{\max}, \forall k \in \mathcal{K}. \end{aligned} \quad (P_1)$$

Note that Problem P_1 is equivalent to Problem P_c when Problem P_c is feasible. On the other hand, the objective function of Problem P_1 is equal to $-\infty$ if and only if Problem P_c is infeasible.

B. Monotonic Optimization

Due to the non-convex nature of Problem P_1 , it is impossible to find the optimal solution based on the theory of convex optimization [13]. Fortunately, it is recently found in operations research that monotonicity is another important property besides convexity that can be exploited to efficiently solve an optimization problem. By exploiting the hidden monotonicity of optimization problems, we can bypass the non-convexity issue and obtain the global optimal solutions efficiently. In this subsection, we first introduce the definition of MO, and then show how Problem P_1 can be transformed into a MO problem. We further discuss several key properties of the reformulation that are critical for the design of the S-MAPEL algorithm that solves the optimization problem.

Definition 1 (Normal): An infinite set $\mathcal{F} \subset \mathbb{R}_+^N$ (the N -dim nonnegative real domain) is said to be normal if for any element $\mathbf{v} \in \mathcal{F}$, the set $[\mathbf{0}, \mathbf{v}] \subset \mathcal{F}$.

Definition 2 (MO): An optimization problem belongs to the class of MO if it can be represented by the following formulation:

$$\begin{aligned} \text{maximize}_{\mathbf{x}} & \Phi(\mathbf{x}) \\ \text{subject to} & \mathbf{x} \in \mathcal{H}, \end{aligned} \quad (P_M)$$

where set $\mathcal{H} \subset \mathbb{R}_+^N$ is a nonempty normal closed set and function $\Phi(\mathbf{x})$ is an increasing function on \mathbb{R}_+^N [12].

Definition 3 (Upper boundary): A point $\mathbf{y} \in \mathbb{R}_+^N$ is an upper boundary point of a bounded normal set \mathcal{F} if $\mathbf{y} \in \mathcal{F}$ while $\{\mathbf{y}' \in \mathbb{R}_+^N | \mathbf{y}' \succ \mathbf{y}\} \subset \mathbb{R}_+^N \setminus \mathcal{F}$ ³. The set of all upper boundary points of \mathcal{F} is the upper boundary of \mathcal{F} .

Proposition 1: The optimal solution of the MO Problem P_M (if it exists) is attained on the upper boundary of \mathcal{G} [12].

Now we are ready to reformulate Problem P_1 into a MO problem.

Let $(\boldsymbol{\delta}, \mathbf{z})$ ⁴ denote the concatenation of two vectors $\boldsymbol{\delta} = (\delta^1, \dots, \delta^{M+1})$ and $\mathbf{z} = (z_1^1, \dots, z_1^M, \dots, z_2^{M+1}, \dots, z_M^{M+1})$. Since the function $\hat{U}_i(r_i)$ is non-decreasing in r_i , it is easy to see that the function $\Phi((\boldsymbol{\delta}, \mathbf{z})) = \sum_{i=1}^M \hat{U}_i(\sum_{k=1}^{M+1} \delta^k \log_2(1 + z_i^k))$ is a non-decreasing function on $\mathbb{R}_+^{(M^2+2M+1)}$. That is, for any two vectors $(\boldsymbol{\delta}_1, \mathbf{z}_1)$ and $(\boldsymbol{\delta}_2, \mathbf{z}_2)$ such that $(\boldsymbol{\delta}_1, \mathbf{z}_1) \succeq (\boldsymbol{\delta}_2, \mathbf{z}_2)$, we have $\Phi((\boldsymbol{\delta}_1, \mathbf{z}_1)) \geq \Phi((\boldsymbol{\delta}_2, \mathbf{z}_2))$. We further note that $\gamma_i(\mathbf{p}^k)$ for all i and k is nonnegative and the time fraction vector $\boldsymbol{\beta}$ is nonnegative as well. Based on these observations, Problem P_1 can be rewritten as

$$\begin{aligned} \text{maximize}_{(\boldsymbol{\delta}, \mathbf{z})} & \Phi((\boldsymbol{\delta}, \mathbf{z})) = \sum_{i=1}^M \hat{U}_i\left(\sum_{k=1}^{M+1} \delta^k \log_2(1 + z_i^k)\right) \\ \text{subject to} & (\boldsymbol{\delta}, \mathbf{z}) \in \mathcal{G}, \end{aligned} \quad (P_2)$$

where the feasible set

$$\mathcal{G} = \left\{ (\boldsymbol{\delta}, \mathbf{z}) \mid 0 \leq \delta^k \leq \beta^k \text{ and } 0 \leq z_i^k \leq \gamma_i(\mathbf{p}^k), \forall i \in \mathcal{M}, \forall k \in \mathcal{K}, (\boldsymbol{\beta}, \mathbf{p}) \in \mathcal{P}_\beta \right\} \quad (8)$$

with

$$\mathcal{P}_\beta = \left\{ (\boldsymbol{\beta}, \mathbf{p}) \mid \sum_{k=1}^{M+1} \beta^k \leq 1, \beta^k \geq 0 \text{ and } 0 \leq p_i^k \leq P_i^{\max}, \forall i \in \mathcal{M}, \forall k \in \mathcal{K} \right\}. \quad (9)$$

Here, $(\boldsymbol{\beta}, \mathbf{p})$ is the concatenation of vectors $\boldsymbol{\beta} = (\beta^1, \dots, \beta^{M+1})$ and $\mathbf{p} = (p_1^1, \dots, p_M^1, \dots, p_1^{M+1}, \dots, p_M^{M+1})$. In the following, we establish the equivalence between Problems P_2 and P_1 .

Definition 4 (Box): Given any vector $\mathbf{v} \in \mathbb{R}_+^N$, the hyper rectangle $[\mathbf{0}, \mathbf{v}] = \{\mathbf{x} | \mathbf{v} \succeq \mathbf{x} \succeq \mathbf{0}\}$ is referred to as a box with vertex \mathbf{v} .

Remark 2: A box is normal.

³In this paper, $\mathcal{A} \setminus \mathcal{B}$ denotes the set $\{\mathbf{x} | \mathbf{x} \in \mathcal{A} \text{ and } \mathbf{x} \notin \mathcal{B}\}$.

⁴In this paper, we use the notation (\mathbf{a}, \mathbf{b}) to represent the concatenation of two vectors \mathbf{a} and \mathbf{b} .

Proposition 2: The intersection and the union of a family of normal sets are normal sets.

The feasible set \mathcal{G} of Problem P_2 is essentially a union of infinite number of boxes with vertices of all boxes belonging to the set $\{(\boldsymbol{\tau}, \mathbf{c}) | \tau^k = \beta^k \text{ and } c_i^k = \gamma_i(\mathbf{p}^k), \forall i \in \mathcal{M}, \forall k \in \mathcal{K}, (\boldsymbol{\beta}, \mathbf{p}) \in \mathcal{P}_\beta\}$. Thus, by Proposition 2, the feasible set is a normal set. This, together with the fact that $\Phi((\boldsymbol{\delta}, \mathbf{z}))$ is an increasing function of $(\boldsymbol{\delta}, \mathbf{z})$, implies that Problem P_2 is a MO problem. Thus, by Proposition 1, the optimal solution of Problem P_2 , denoted by $(\boldsymbol{\delta}^*, \mathbf{z}^*)$, must occur at the upper boundary of set \mathcal{G} where $\delta^k = \beta^k$ and $z_i^k = \gamma_i(\mathbf{p}^k)$ for all i and k . If we can find a joint time fraction and power allocation $(\boldsymbol{\beta}^*, \mathbf{p}^*)$ corresponding to the optimal solution $(\boldsymbol{\delta}^*, \mathbf{z}^*)$ such that $\delta^{k*} = \beta^{k*}$ and $z_i^{k*} = \gamma_i(\mathbf{p}^{k*})$ for all i and k , then such $(\boldsymbol{\beta}^*, \mathbf{p}^*)$ is clearly the optimal solution to Problem P_1 . Finding such $(\boldsymbol{\beta}^*, \mathbf{p}^*)$ requires solving $M+1$ uncoupled sets of M linear equations $z_i^{k*} (\sum_{j \neq i} G_{ji} p_i^{k*} + n_i) = G_{ii} p_i^{k*}$ with

M variables $p_1^{k*}, \dots, p_M^{k*}$. As the channel gains G_{ij} 's are generally random, we can show with probability 1 that the $M^2 + M$ equations are linearly independent, implying that there is a unique solution $(\mathbf{p}^{1*}, \dots, \mathbf{p}^{M+1*})$. Hence, Problems P_c, P_1 and P_2 are all equivalent with each other. *We will focus on how to solve Problem P_2 efficiently based on the recent advance in monotonic optimization in the rest of the paper.*

Before leaving this section, note that set \mathcal{G} is a non-convex set. However, convexity is not important in obtaining the global optimal solution. In the next section, we show that it is the monotonicity of the objective function and the normality of the feasible set in the reformulated Problem P_2 that facilitates efficient calculation of the global optimal solution.

IV. THE S-MAPEL ALGORITHM

In this section, we propose a novel algorithm, S-MAPEL, to solve Problem P_2 based on the special characteristics of the problem. We first review the general MO algorithm before presenting the algorithm for joint power control and scheduling.

A. General MO Algorithm

For the paper to be self-contained, some relevant definitions are first presented before we introduce the general MO algorithm.

Definition 5 (Polyblock): Given any finite set $\mathcal{T} \subset \mathbb{R}_+^N$ with elements \mathbf{v}_i , the union of all the boxes $[\mathbf{0}, \mathbf{v}]$ is referred to as a polyblock with vertex set \mathcal{T} .

Definition 6 (Proper): An element $\mathbf{v} \in \mathcal{T}$ is proper if there does not exist $\tilde{\mathbf{v}} \in \mathcal{T}$, such that $\mathbf{v} \neq \tilde{\mathbf{v}}$ and $\tilde{\mathbf{v}} \succeq \mathbf{v}$. If every element $\mathbf{v} \in \mathcal{T}$ is proper, then the set \mathcal{T} is a proper set.

Definition 7 (Projection): Given any nonempty normal set $\mathcal{F} \subset \mathbb{R}_+^N$ and any $\mathbf{v} \in \mathbb{R}_+^N \setminus \{\mathbf{0}\}$, $\boldsymbol{\pi}^{\mathcal{F}}(\mathbf{v})$ is a projection of \mathbf{v} on \mathcal{F} if $\boldsymbol{\pi}^{\mathcal{F}}(\mathbf{v}) = \lambda \mathbf{v}$ with $\lambda = \max\{\kappa | \kappa \mathbf{v} \in \mathcal{F}\}$. In other words, $\boldsymbol{\pi}^{\mathcal{F}}(\mathbf{v})$ is the unique point where the halfline from $\mathbf{0}$ through \mathbf{v} meets the upper boundary of \mathcal{F} .

Based on the above concepts, an algorithm that solves a general MO problem P_M works as follows. We first construct a polyblock \mathcal{S}_1 that contains the feasible set \mathcal{H} of Problem

P_M . Let \mathcal{T}_1 denotes the proper vertex set of \mathcal{S}_1 . By Proposition 2 of [2], the maximum of the objective function of Problem P_M (i.e., $\Phi(\mathbf{x})$) over set \mathcal{S}_1 occurs at some proper vertex \mathbf{x}_1 of \mathcal{S}_1 , i.e., $\mathbf{x}_1 \in \mathcal{T}_1$. If \mathbf{x}_1 happens to reside in set \mathcal{H} as well, then it solves Problem P_M and the optimal solution \mathbf{x}^* is equal to \mathbf{x}_1 . Otherwise, based on Proposition 3 of [2], we can construct a smaller polyblock $\mathcal{S}_2 \subset \mathcal{S}_1$ that still contains \mathcal{H} but excludes \mathbf{x}_1 . This is achieved by constructing the vertex set \mathcal{T}_2 by replacing \mathbf{x}_1 in \mathcal{T}_1 with N new vertices $\{\mathbf{x}_{11}, \dots, \mathbf{x}_{1N}\}$ and removing improper vertices, where $\mathbf{x}_{1j} = \mathbf{x}_1 - (x_{1,j} - \pi_j^{\mathcal{H}}(\mathbf{x}_1))\mathbf{e}_j$. We can repeat this procedure until an optimal solution is found. This leads to a sequence of polyblocks containing $\mathcal{H} : \mathcal{S}_1 \supset \mathcal{S}_2 \supset \dots \supset \mathcal{H}$. Obviously, $\Phi(\mathbf{x}_1) \geq \Phi(\mathbf{x}_2) \geq \dots \geq \Phi(\mathbf{x}^*)$, where \mathbf{x}_n is the optimal solution that maximizes $\Phi(\mathbf{x})$ over the polyblock \mathcal{S}_n . The algorithm terminates at n th iteration if $\mathbf{x}_n \in \mathcal{H}$. This general algorithm is guaranteed to converge to a global optimal solution *only if* the optimal solution \mathbf{x}^* has a positive lower bound \mathbf{x}_l (i.e., $\mathbf{x}_l \succ \mathbf{a}$, where $\mathbf{a} > \mathbf{0}$) [12] [14].

The MAPEL algorithm proposed in our earlier paper [2] is one of the earliest successful applications of MO to engineering designs in wireless networks. The MO problem formulated in [2] has a nice property that the optimal solution is lower bounded by a positive value, which guarantees the convergence of MAPEL to the global optimal solution. On the contrary, Problem P_2 in this paper does not impose a positive lower bound on the optimal solution $(\boldsymbol{\delta}^*, \mathbf{z}^*)$. In fact, some δ^{k*} 's and z_i^{k*} 's can be equal to zero at the optimal solution. Consequently, the general MO algorithm described above (and hence MAPEL) cannot be directly applied even though Problem P_2 is a MO problem. In the next subsection, we propose an enhanced algorithm, referred to as S-MAPEL, to obtain the global optimal solution of P_2 through non-trivial modifications of the general MO algorithm.

B. The S-MAPEL Algorithm

In the general MO algorithm, a critical step to construct new polyblocks is calculating the projection $\boldsymbol{\pi}^{\mathcal{H}}(\mathbf{x}_n)$ as defined in Definition 7. To regain the convergence that was destroyed due to the absence of a positive lower bound on the optimal solution to P_2 , S-MAPEL adopts a modified projection $\boldsymbol{\pi}_o(\cdot)$ where a vertex is projected onto the upper boundary of the feasible set along a line connecting the vertex and a negative point $\mathbf{o} \prec \mathbf{0}$. To make sure that the projection always exists, the projection region \mathcal{G} is extended to a new region $\mathcal{G}' = \{(\boldsymbol{\delta}', \mathbf{z}') | \mathbf{o} \preceq (\boldsymbol{\delta}', \mathbf{z}') \preceq (\boldsymbol{\delta}, \mathbf{z}), \forall (\boldsymbol{\delta}, \mathbf{z}) \in \mathcal{G}\}$ without sacrificing the optimality of P_2 . Mathematically, the modified projection is $\boldsymbol{\pi}_o^{\mathcal{G}'}((\boldsymbol{\delta}_n, \mathbf{z}_n)) = \lambda_n((\boldsymbol{\delta}_n, \mathbf{z}_n) - \mathbf{o}) + \mathbf{o}$, where $\lambda_n = \max\{\lambda | \lambda_n((\boldsymbol{\delta}_n, \mathbf{z}_n) - \mathbf{o}) + \mathbf{o} \in \mathcal{G}'\}$. This is illustrated in Fig. 4. Without loss of generality, set $\mathbf{o} = -\mathbf{1}$ hereafter. Note that the optimal solution $(\boldsymbol{\delta}^*, \mathbf{z}^*)$ to P_2 is always component-wise larger than or equal to $\mathbf{0}$. This implies that the optimal solution $(\boldsymbol{\delta}^*, \mathbf{z}^*)$ has a ‘‘positive’’ lower bound with respect to the ‘‘new origin’’ $-\mathbf{1}$. This crucial modification guarantees the convergence of S-MAPEL, as we will prove in subsection IV.C.

The projection $\boldsymbol{\pi}_{-\mathbf{1}}^{\mathcal{G}'}((\boldsymbol{\delta}_n, \mathbf{z}_n)) = \lambda_n((\boldsymbol{\delta}_n, \mathbf{z}_n) + \mathbf{1}) - \mathbf{1}$ is

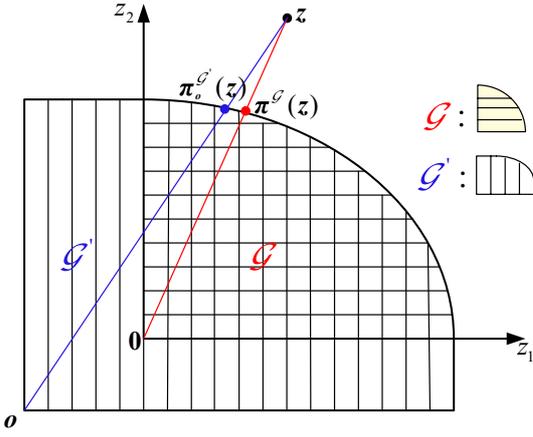


Fig. 4. The modified projection $\pi_0^{G'}(z)$.

obtained by solving the following max-min problem for λ_n :

$$\begin{aligned} \lambda_n &= \max \left\{ \lambda \mid \lambda((\delta_n, z_n) + \mathbf{1}) - \mathbf{1} \in \mathcal{G}' \right\} \\ &= \max \left\{ \lambda \mid \lambda \leq \min_{i \in \mathcal{M}, k \in \mathcal{K}} \left\{ A_i(\mathbf{p}^k), \frac{1 + \beta^k}{1 + \delta_n^k} \right\}, (\beta, \mathbf{p}) \in \mathcal{P}_\beta \right\} \\ &= \max_{(\beta, \mathbf{p}) \in \mathcal{P}_\beta} \min_{i \in \mathcal{M}, k \in \mathcal{K}} \left\{ A_i(\mathbf{p}^k), \frac{1 + \beta^k}{1 + \delta_n^k} \right\} \\ A_i(\mathbf{p}^k) &= \frac{1 + \gamma_i(\mathbf{p}^k)}{1 + z_{n,i}^k}. \end{aligned} \quad (10)$$

This is known as a generalized fractional linear programming problem and can be directly solved by the Dinkelbach-type algorithm [15] with slight modification. For the max-min problem (10), the Dinkelbach-type algorithm is a serial algorithm involving $(M + 1)^2$ variables. A close look at (10) indicates that the terms in the max-min operation are uncoupled functions of either β or \mathbf{p}^k 's, but not both. Hence, we can further write (10) as follows.

$$\begin{aligned} \lambda_n &= \max_{(\beta, \mathbf{p}) \in \mathcal{P}_\beta} \min \left\{ \min_{i \in \mathcal{M}} \left\{ \frac{1 + \gamma_i(\mathbf{p}^1)}{1 + z_{n,i}^1} \right\}, \dots, \right. \\ &\quad \left. \min_{i \in \mathcal{M}} \left\{ \frac{1 + \gamma_i(\mathbf{p}^{M+1})}{1 + z_{n,i}^{M+1}} \right\}, \min_{k \in \mathcal{K}} \left\{ \frac{1 + \beta^k}{1 + \delta_n^k} \right\} \right\} \\ &= \min \left\{ \max_{\mathbf{p}^1 \in [0, \mathbf{P}^{\max}]} \min_{i \in \mathcal{M}} \frac{1 + \gamma_i(\mathbf{p}^1)}{1 + z_{n,i}^1}, \dots, \right. \\ &\quad \left. \max_{\mathbf{p}^{M+1} \in [0, \mathbf{P}^{\max}]} \min_{i \in \mathcal{M}} \frac{1 + \gamma_i(\mathbf{p}^{M+1})}{1 + z_{n,i}^{M+1}}, \right. \\ &\quad \left. \max_{\sum_{k=1}^{M+1} \beta^k \leq 1, \beta \geq 0} \min_{k \in \mathcal{K}} \frac{1 + \beta^k}{1 + \delta_n^k} \right\} \\ &= \min \left\{ \lambda_n^1, \dots, \lambda_n^{M+1}, \lambda_n^\beta \right\}, \end{aligned} \quad (11)$$

where $\lambda_n^k = \max_{\mathbf{p}^k \in [0, \mathbf{P}^{\max}]} \min_{i \in \mathcal{M}} \frac{1 + \gamma_i(\mathbf{p}^k)}{1 + z_{n,i}^k}$ for $k = 1, \dots, M + 1$, and $\lambda_n^\beta = \max_{\sum_{k=1}^{M+1} \beta^k \leq 1, \beta \geq 0} \min_{k \in \mathcal{K}} \frac{1 + \beta^k}{1 + \delta_n^k}$. Eqn. (11) indicates that the

calculation of λ_n can be decomposed into calculating λ_n^k 's and λ_n^β . This suggests the possibility of a parallel algorithm for fast computation. In particular, each λ_n^k can be obtained through the Dinkelbach-type algorithm with M variables p_i^k 's, and λ_n^β can be obtained through Dinkelbach-type algorithm with $M + 1$ variables β^k 's. Since λ_n^k 's and λ_n^β are all the generalized fractional linear programming, their calculations are polynomial time solvable [15]. The calculations of λ_n^k 's and λ_n^β are presented in Algorithm 1, where we have focused on an arbitrary λ_n^k without loss of generality.

Having calculated λ_n^k 's and λ_n^β through Dinkelbach-type Algorithm (i.e., Algorithm 1), we can obtain the projection $\pi_{-1}^{G'}((\delta_n, z_n)) = \lambda_n((\delta_n, z_n) + \mathbf{1}) - \mathbf{1}$ with $\lambda_n = \min\{\lambda_n^1, \dots, \lambda_n^{M+1}, \lambda_n^\beta\}$.

Algorithm 1 Dinkelbach-type Algorithm (for finding λ_n^k)

- 1: **Initialization:** Choose $\mathbf{p}^{k(0)} \in [0, \mathbf{P}^{\max}]$ and let $m = 0$.
 - 2: **repeat**
 - 3: Given $\mathbf{p}^{k(m)}$, solve $\lambda_n^{k(m)} = \min_{1 \leq i \leq M} \frac{1 + \gamma_i^k(\mathbf{p}^{k(m)})}{1 + z_{n,i}^k}$.
 - 4: Given $\lambda_n^{k(m)}$, solve

$$\mathbf{p}^{k(m+1)} = \operatorname{argmax}_{\mathbf{p}^k \in [0, \mathbf{P}^{\max}]} \min_{i \in \mathcal{M}} \left(\left(\sum_{j=1}^M G_{ji} p_j^k + n_i \right) - \lambda_n^{k(m)} (z_{n,i}^k + 1) \left(\sum_{j \neq i} G_{ji} p_j^k + n_i \right) \right)$$
 for all k .
 - 5: $m = m + 1$.
 - 6: **until** $\min_{i \in \mathcal{M}} \left(\left(\sum_{j=1}^M G_{ji} p_j^{k(m-1)} + n_i \right) - \lambda_n^{k(m-1)} (z_{n,i}^k + 1) \left(\sum_{j \neq i} G_{ji} p_j^{k(m-1)} + n_i \right) \right) \leq 0$.
 - 7: $\lambda_n^k = \lambda_n^{k(m-1)}$.
-

With the modified projection, the execution of the S-MAPEL largely follows the general MO algorithm. Specifically, in the n th iteration, the optimal vertex (δ_n, z_n) is selected from the vertex set \mathcal{T}_n of the outer polyblock \mathcal{S}_n through $(\delta_n, z_n) = \operatorname{argmax}\{\Phi((\delta, z)) \mid (\delta, z) \in \mathcal{T}_n\}$. Then, the new polyblock \mathcal{S}_{n+1} is obtained by replacing (δ, z) in \mathcal{T}_n with $(M + 1)^2$ new vertices $\{(\delta_{n1}, z_{n1}), \dots, (\delta_{n(M+1)2}, z_{n(M+1)2})\}$, where $(\delta_{nj}, z_{nj}) = (\delta_n, z_n) - ((\delta_n, z_n)_j - \pi_{-1}^{G'}((\delta_n, z_n))e_j)$, and removing improper vertices and all vertices (δ, z) that do not satisfy $(\delta, z) \succeq \mathbf{0}$.

S-MAPEL terminates at the n th iteration if $(\delta_n, z_n) \in \mathcal{G}$. In practice, we say $(\delta_n, z_n) \in \mathcal{G}$ when $(1 + \epsilon)\Phi((\delta'_n, z'_n)) \geq \Phi((\delta_n, z_n))$, where $\epsilon > 0$ is a small positive number representing the error tolerance level, and $(\delta'_n, z'_n) \in \mathcal{G}$ is the current best feasible solution (CBS) that is known so far. Such a stopping criteria can guarantee to obtain an ϵ -optimal solution [2]. In particular, CBS is updated in each iteration as $(\delta'_n, z'_n) = \operatorname{argmax}\{\Phi(\mathbf{y}) \mid \mathbf{y} \in \{\pi_{-1}^{G'}((\delta'_n, z'_n)), (\delta'_{n-1}, z'_{n-1})\}\}$ if $\pi_{-1}^{G'}((\delta_n, z_n)) \succeq \mathbf{0}$, and $(\delta'_n, z'_n) = (\delta'_{n-1}, z'_{n-1})$ otherwise. The corresponding objective function value $\Phi((\delta'_n, z'_n))$ is referred to as the current best value (CBV). For initialization, $(\delta'_1, z'_1) = \pi_{-1}^{G'}((\delta_1, z_1))$ if $\pi_{-1}^{G'}((\delta_1, z_1)) \succeq \mathbf{0}$, and $(\delta'_1, z'_1) = \mathbf{0}$ otherwise. Obviously, we have $\Phi((\delta'_1, z'_1)) \leq \Phi((\delta'_2, z'_2)) \leq \dots \leq \Phi((\delta^*, z^*))$.

Having introduced the basic operations, we now formally present S-MAPEL in Algorithm 2.

Algorithm 2 The S-MAPEL Algorithm

1: **Initialization:** Choose the error tolerance $\epsilon > 0$, and let $n = 1$.

2: **repeat**

3: If $n = 1$, construct the initial polyblock \mathcal{S}_1 with vertex set $\mathcal{T}_1 = \{(\mathbf{b}, \mathbf{v})\}$, where $b^k = 1$ for all k , and

$$v_i^k = \max_{\mathbf{p}^k \in [0, \mathbf{P}^{\max}]} \gamma_i(\mathbf{p}^k) = \frac{G_{ii} P_i^{\max}}{n_i}, \forall i \in \mathcal{M}, \forall k \in \mathcal{K}.$$

It is clear that polyblock \mathcal{S}_1 is a box $[\mathbf{0}, (\mathbf{b}, \mathbf{v})]$ containing \mathcal{G} . If $n > 1$, construct a smaller polyblock \mathcal{S}_n with vertex set \mathcal{T}_n by replacing $(\delta_{n-1}, \mathbf{z}_{n-1})$ in \mathcal{T}_{n-1} with $(M+1)^2$ new vertices $\{(\delta_{n-11}, \mathbf{z}_{n-11}), \dots, (\delta_{n-1(M+1)^2}, \mathbf{z}_{n-1(M+1)^2})\}$ and removing improper vertices and all vertices (δ, \mathbf{z}) which do not satisfy $(\delta, \mathbf{z}) \succeq \mathbf{0}$. Herein, $(\delta_{n-1j}, \mathbf{z}_{n-1j}) = (\delta_{n-1}, \mathbf{z}_{n-1}) - ((\delta_{n-1}, \mathbf{z}_{n-1})_j - \pi_{-1j}^{\mathcal{G}'((\delta_{n-1}, \mathbf{z}_{n-1}))})e_j$,

4: Find (δ_n, \mathbf{z}_n) that maximizes the objective function of Problem P_2 over set \mathcal{T}_n , i.e.,

$$(\delta_n, \mathbf{z}_n) = \operatorname{argmax}\{\Phi((\delta, \mathbf{z})) | (\delta, \mathbf{z}) \in \mathcal{T}_n\}, \quad (12)$$

5: Find $\pi^{\mathcal{G}'((\delta_n, \mathbf{z}_n))}$ based on Algorithm 2.

6: Determine CBS $(\delta'_n, \mathbf{z}'_n)$ and CBV $\Phi((\delta'_n, \mathbf{z}'_n))$, where $(\delta'_n, \mathbf{z}'_n) = \operatorname{argmax}\{\Phi(\mathbf{y}) | \mathbf{y} \in \{\pi_{-1}^{\mathcal{G}'((\delta_n, \mathbf{z}_n))}, (\delta'_{n-1}, \mathbf{z}'_{n-1})\}\}$ if $\pi_{-1}^{\mathcal{G}'((\delta_n, \mathbf{z}_n))} \succeq \mathbf{0}$, and $(\delta'_n, \mathbf{z}'_n) = (\delta'_{n-1}, \mathbf{z}'_{n-1})$ otherwise. For initialization, $(\delta'_1, \mathbf{z}'_1) = \pi_{-1}^{\mathcal{G}'((\delta_1, \mathbf{z}_1))}$ if $\pi_{-1}^{\mathcal{G}'((\delta_1, \mathbf{z}_1))} \succeq \mathbf{0}$, and $(\delta'_1, \mathbf{z}'_1) = \mathbf{0}$ otherwise.

7: $n = n + 1$.

8: **until** $(1 + \epsilon)\Phi((\delta'_{n-1}, \mathbf{z}'_{n-1})) \geq \Phi((\delta_{n-1}, \mathbf{z}_{n-1}))$.

9: If $\Phi((\delta'_{n-1}, \mathbf{z}'_{n-1})) > -\infty$, then compute the optimal time fraction and power allocation (β^*, \mathbf{p}^*) (i.e., optimal solution to Problem P_c) by solving $\delta_{n-1}^{k'} = \beta^{k^*}$ and $z_{n-1,i}^{k'} = \gamma_i(\mathbf{p}^{k^*})$ for all i and k . Otherwise, there is no feasible solution of Problem P_c .

C. Global Convergence

With the modified projection $\pi_{\mathcal{O}}(\cdot)$, Theorem 2 shows that S-MAPEL converges to the optimal solution of Problem P_2 .

Theorem 2: The S-MAPEL algorithm globally converges to a global optimal solution of Problem P_2 .

The proof is relegated to Appendix-A.

Before leaving this subsection, note that although S-MAPEL is proved to converge to the global optimal solution, the convergence speed is still an open problem. It is, however, proved in [12] that a MO algorithm is guaranteed to converge within finite iterations as long as $\epsilon > 0$. An advantage of the S-MAPEL algorithm is that we can trade off performance for convergence time by tuning ϵ . The smaller ϵ , the longer the algorithm runs and the more accurate the optimal solution is.

V. AN ACCELERATED ALGORITHM FOR JOINT POWER CONTROL AND SCHEDULING

In S-MAPEL, the size of the vertex set \mathcal{T}_n grows quickly in each iteration when M is large. This could potentially lead to long convergence time, as a best vertex has to be selected by comparing all vertices in \mathcal{T}_n in each iteration. Such a problem is not avoidable if the global optimal solution is to be guaranteed, as Problem P_2 is essentially NP-hard. In this section, we present an accelerated algorithm A-S-MAPEL (where the prefix ‘‘A’’ stands for accelerated) that prevents the size of the vertex set from growing quickly with each iteration. By doing so, the convergence of the algorithm is drastically expedited. Our numerical results will show that the gap between the solution obtained by A-S-MAPEL and the global optimal solution is negligible.

The intuition behind A-S-MAPEL algorithm is as follows. Consider an optimal solution to Problem P_2 $(\delta^*, \mathbf{z}^*) = (\delta^{1*}, \dots, \delta^{M+1*}, z_1^{1*}, \dots, z_M^{1*}, \dots, z_1^{M+1*}, \dots, z_M^{M+1*})$. A close look at Problem P_2 suggests that a new vector obtained by swapping the values of $(\delta^{i*}, z_1^{i*}, \dots, z_M^{i*})$ and $(\delta^{j*}, z_1^{j*}, \dots, z_M^{j*})$ for any pair of i and j is also an optimal solution. This is because the ordering of the time segments does not affect the sum data rate of each user, and hence does not affect the value of the utility functions. This inherent symmetry implies that there could exist more than one equally optimal polyblock vertex (δ_n, \mathbf{z}_n) at each iteration of the S-MAPEL algorithm. Specifically, these equally optimal vertices would lead to the same optimality just with different ordering for the time segments. Therefore, selecting any one of these vertices at each iteration while eliminating the others would not affect the optimality of the algorithm.

One difficulty in carrying out this idea lies in the identification of symmetric vertices from all vertices that yield the same optimal value at an iteration. To address this issue, A-S-MAPEL makes a simplifying assumption that all equally optimal vertices are symmetric vertices. Denoting the equally optimal vertices at the n th iteration as $\mathcal{Z}_n = \{(\delta, \mathbf{z}) | \Phi((\delta, \mathbf{z})) = \Phi((\tilde{\delta}, \tilde{\mathbf{z}})) \text{ and } (\delta, \mathbf{z}) \in \mathcal{T}_n\}$, where $(\tilde{\delta}, \tilde{\mathbf{z}}) = \operatorname{argmax}\{\Phi((\delta, \mathbf{z})) | (\delta, \mathbf{z}) \in \mathcal{T}_n\}$. A-S-MAPEL then selects the one with the smallest difference between $\Phi((\delta, \mathbf{z}))$ and $\Phi(\pi_{-1}^{\mathcal{G}'((\delta, \mathbf{z}))})$, and delete all other vertices in \mathcal{Z}_n .

For practical implementation, we allow the existence of an error tolerance tol to further expedite the computational speed. Thus, the set \mathcal{Z}_n is extended to

$$\mathcal{Z}_n = \{(\delta, \mathbf{z}) | (1 + tol) \times \Phi((\delta, \mathbf{z})) \geq \Phi((\tilde{\delta}, \tilde{\mathbf{z}})) \text{ and } (\delta, \mathbf{z}) \in \mathcal{T}_n\}. \quad (13)$$

With the above notions, A-S-MAPEL is the same as S-MAPEL except for Step 4, which is modified as follows.

Step 4. Select (δ_n, \mathbf{z}_n) according to

$$(\delta_n, \mathbf{z}_n) = \operatorname{argmin}\{\Phi((\delta, \mathbf{z})) - \Phi(\pi_{-1}^{\mathcal{G}'((\delta, \mathbf{z}))}) | (\delta, \mathbf{z}) \in \mathcal{Z}_n\}, \quad (14)$$

and delete all other vertices in \mathcal{Z}_n .

Similar to S-MAPEL algorithm, an advantage of the A-S-MAPEL algorithm is that we can trade off performance for convergence time by tuning ϵ and tol . The smaller ϵ and tol , the longer the algorithm runs and the more accurate the

optimal solution is. In addition, it is due to the tolerance tol and the assumption that all equally optimal vertices are symmetric vertices that A-S-MAPEL is a suboptimal method instead of an optimal one.

VI. PERFORMANCE EVALUATION

A. Near Optimality of A-S-MAPEL

We first consider a four-link network as shown in Fig. 5, where $d = 5$ meters. Assume the channel gain between the transmitter node i and the receiver node j is d_{ij}^{-4} , where d_{ij} denotes the distance between the two nodes. Assume that $\mathbf{P}^{\max} = (1.0 \ 1.0 \ 1.0 \ 1.0)\text{mW}$, and $n_i = 0.1\mu\text{W}$ for all links. There are no minimum data rate constraints in this example. In Fig. 6 and Fig. 8, we investigate the system utility obtained by A-S-MAPEL under different error tolerances ϵ and tol when $U_i(r_i) = \log r_i$ and $U_i(r_i) = \frac{1}{1+\exp(-r_i+2)}$, respectively. For comparison, the optimal system utilities are also plotted. Correspondingly, the numbers of iterations for convergence are plotted in Fig. 7 and Fig. 9, respectively.

From Figs. 6 and 8, it can be seen that the A-S-MAPEL algorithm can perform very close to the global optimal solution. For example, when $tol = 0.0005$, A-S-MAPEL obtains a system utility that is only 0.33% away from the optimum for proportional fair utility, and 0.17% away from the optimum for sigmoidal utility. On the other hand, it is not surprising to see that the algorithm performance improves with the decrease of either ϵ or tol . In general, the algorithm performance is not sensitive to the value of ϵ when tol is small enough. For example, when $tol = 0.0005$, the obtained system utility at any $\epsilon \in [0, 0.5]$ is the same for both proportional fair utility and sigmoidal utility. This observation illustrates the algorithm performance can be guaranteed as long as choosing small enough ϵ and tol . It can be seen from Figs. 7 and 9 that the convergence time of A-S-MAPEL can be quite different for different objective functions. Regardless, the total number of iterations needed increases when either ϵ or tol decreases, and the change is drastic when tol is close to 0. Obviously, parameters ϵ and tol provide a tuning knob for achieving various trade-off between algorithm performance and convergence time.

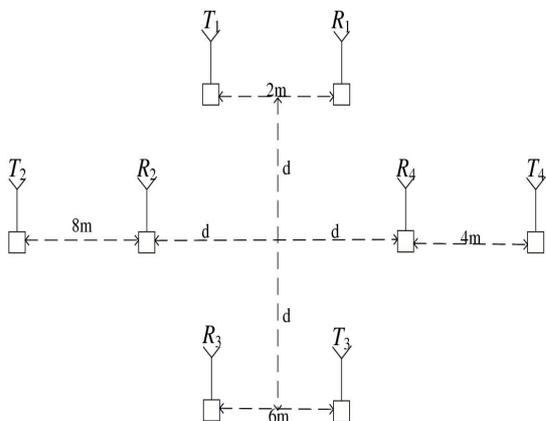


Fig. 5. A network topology with four links.

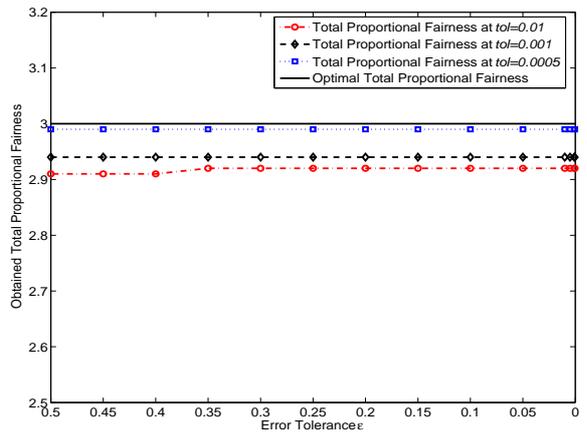


Fig. 6. Obtained total proportional fairness for different error tolerance ϵ .

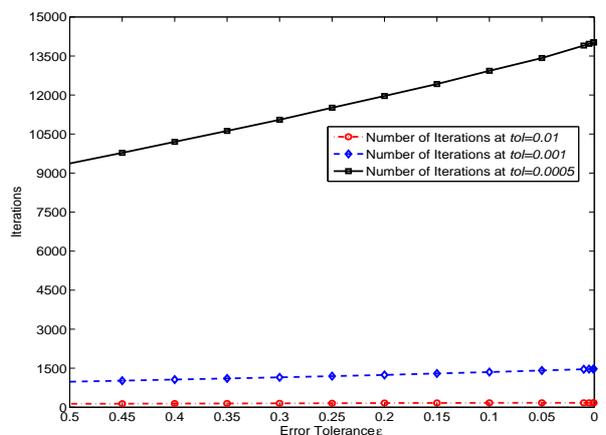


Fig. 7. The number of iterations needed for different error tolerance ϵ when obtaining total proportional fairness.

B. Optimal Joint Power Control and Scheduling vs. Node Density

In this subsection, we vary d in Fig. 5 to investigate the effect of node density on the optimal power control scheduling. As an example, we set $U_i(r_i) = \log r_i$, and let the minimum data rate constraints be 1.0bps/Hz for all links. Other settings are the same as the previous example. In Fig. 10, we let $d = 5, 10, 15$ meters, and set the scheduling period to be 10 seconds for each d .

It is not surprising to see from Fig. 10 that the optimal power and scheduling heavily depends on the node density. Specifically, when the four links are close to each other (e.g., $d = 5\text{m}$, from 0 to 10 seconds), the optimal transmission power varies with time, implying that the scheduling is an indispensable component in dense networks. On the other hand, scheduling is no longer necessary when the node density is small. For example, when $d = 10\text{m}$ (i.e., from 10 to 20 seconds), the optimal transmission power does not vary with time any more. Furthermore, when links are enough far away from each other (e.g., $d = 15\text{m}$, from 20 to 30 seconds), it is optimal to have all links transmit at the maximum power at the same time.

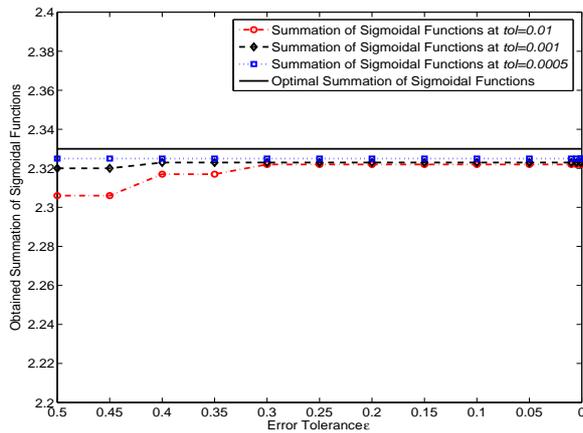


Fig. 8. Obtained summation of sigmoidal functions for different error tolerance ϵ .

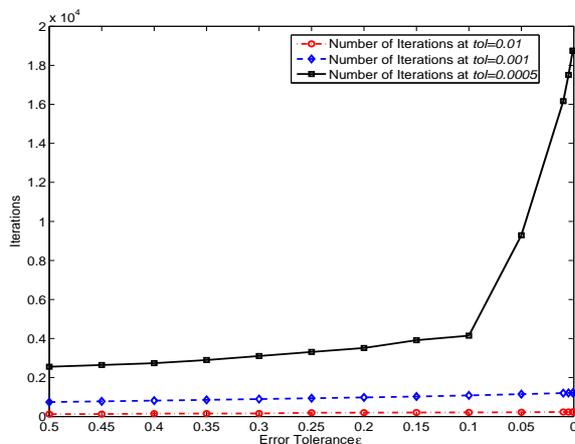


Fig. 9. The number of iterations needed for different error tolerance ϵ when obtaining summation of sigmoidal functions.

C. Performance Study of On-off Power Control, Pure Power Control and On-off Scheduling

One key application of S-MAPEL and A-S-MAPEL is to provide a benchmark to evaluate the performance of other schemes. As an illustration, we evaluate the performance of three widely accepted schemes in the literature, namely pure power control, on-off power control, and on-off power control with scheduling (also referred to as on-off scheduling). In particular, MAPEL [2] is used to obtain the optimal power control solution. With on-off power control, each transmitter either transmits at the maximum power level P_i^{\max} or does not transmit at all. Meanwhile, on-off scheduling is the same as joint power control and scheduling except that transmitters either transmit at the maximum power P_i^{\max} or do not transmit at all. It can be seen that Theorem 1 also applies to this case, and hence no more than $M + 1$ slots are needed to achieve the optimal performance of on-off scheduling.

We consider a collection of n -link networks⁵. Links are

⁵In this paper, on-off scheduling with scheduling is calculated through exhaustive search with $(2^M - 1)^{M+1}$ convex programming. Thus, the calculation is unavailable when the number of links increases, which is why we only consider the network with 3 links or 4 links for on-off scheduling.

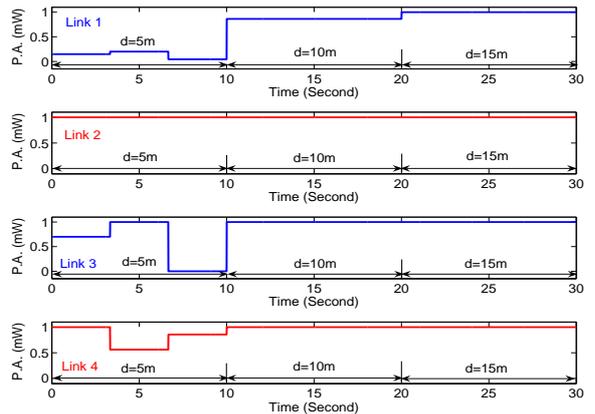


Fig. 10. The optimal allocation vs. node density (P.A.: Power Allocation).

randomly placed in a 15m-by-15m area. The length of each link is uniformly distributed within [1m, 2m]. Meanwhile, set $P_i^{\max}=1\text{mW}$ and $n_i = 0.1\mu\text{W}$. In Table II, the performance of optimal joint power control and scheduling, pure power control, on-off power control, and on-off scheduling are given for different utility functions when $n = 3$ and $n = 4$. Besides, the performance of joint power control and scheduling and pure power control can also be found in Fig. 11 when n is from 3 to 8. Each value in Table II and Fig. 11 is an average over 50 different topologies.

It is not surprising to see that without scheduling, both power control schemes are outperformed by the ones with scheduling. This is because in a dense network, power control alone is not sufficient to eliminate strong levels of interference between close-by links. Interestingly, Fig. 11 shows that the performance gap between joint power control and scheduling and pure power control is wider for $U_i(r_i) = \log r_i$ than it is for $U_i(r_i) = \frac{1}{1+\exp(-r_i+2)}$ when n grows large. This is because for concave utility functions, the derivative of $U_i(r_i)$ is larger for smaller r_i . Hence, the pure power control scheme would force all links to be active but with a low data rate when the network is dense. In this case, scheduling can play an important role in improving the overall system utility. On the other hand, with sigmoidal utility function, the optimal strategy in dense networks is to deactivate some links so that the data rate of other links can exceed the threshold b_i (i.e., 2 in Fig. 11), no matter whether scheduling is employed or not. In this case, scheduling does not make a big difference.

Another interesting observation is that without scheduling, on-off power control may lead to a much lower system utility compared with the optimal power control solution. In contrast, the performance gap between on-off scheduling and optimal joint power control and scheduling is negligible. This is due to the fact that the links that are scheduled to transmit in the same time slot are typically far from each other and do not impose excessive interference on one another. As a result, it is likely to be optimal for the links to transmit at the maximum power level. In practice, most off-the-shelf wireless devices are only allowed to either transmit at the maximum power (i.e., be on) or remain silent (i.e., be off). Therefore, scheduling is

an indispensable component for system utility maximization if “off-the-shelf” wireless devices are to be used. We conclude this section by noting that the design of efficient algorithms for optimal on-off scheduling is more challenging than that for joint power control and scheduling, due to the combinatorial nature of the on-off scheduling problem. It will be part of our future work to design efficient algorithms to solve P_c when only two power levels (P^{\max} and 0) are allowed.

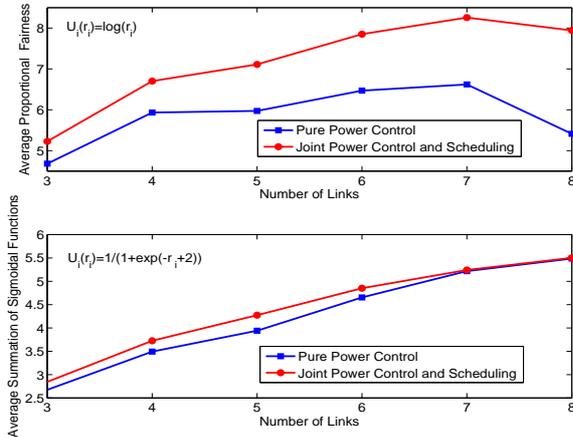


Fig. 11. Average system performance of pure power control vs. joint power control and scheduling in n -link networks.

VII. CONCLUSION

In this paper, we have proposed the S-MAPEL algorithm that efficiently solves the joint power control and scheduling problem in wireless networks. S-MAPEL is guaranteed to converge to an global optimal solution despite the nonconvexity of the problem. The key idea behind the algorithm is to reformulate the non-convex problem into a MO problem, and then construct a sequence of shrinking polyblocks that approximate the upper boundary of the feasible region with increasing precision. We have also established a convenient tradeoff between performance and convergence time of the algorithm. By exploiting the inherent symmetry of the optimal solutions, an accelerated algorithm, A-S-MAPEL, has been proposed.

Guaranteed to converge to the global optimal solution, S-MAPEL provides an important benchmark for performance evaluation of other joint power control and scheduling heuristics in this area. Using this benchmark, we find that the performance gap between on-off scheduling and joint power control and scheduling is negligible. This implies that scheduling is a crucial component in wireless system design, if off-the-shelf wireless devices that only support binary transmit power levels (on or off) are to be used.

There has been a lot of recent work using the primal/dual decomposition approach to devise low-complexity and distributed algorithms for optimal resource allocation problems. These algorithms are guaranteed to converge to the global optimal solutions as long as the optimization problem satisfies the slaters condition [5], [11], [17], [18]. Lemma 1 in this paper suggests that Problem P_c is convex in terms of average

data rate \mathbf{r} . Recently, there have been emerging interests in solving similar problems using primal/dual decomposition, such as [17], [18]. However, we observe that the primal/dual decomposition fails to achieve the global optimal solution of P_c although it is convex in \mathbf{r} . We also find that the primal decomposition can be used to obtain a local optimal solution to Problem P_c in polynomial time. On the other hand, the dual decomposition is not guaranteed to converge. We have omitted the detailed discussions in this paper due to the page limit. Interested readers are referred to the technical report [16].

In our future work, variants of the algorithms will be developed to expedite the convergence and reduce the computational complexity. Moreover, it would be an interesting future research to extend the current work to a multihop network.

APPENDIX

A. Proof of the Theorem 2

Proof: The S-MAPEL algorithm generates a sequence $\{(\delta_n, \mathbf{z}_n)\}$ for $n = 1, 2, \dots$. Each component is calculated as (12) for a newly constructed polyblock. We can find a subsequence $\{(\delta_{n_l}, \mathbf{z}_{n_l})\}$ within the sequence $\{(\delta_n, \mathbf{z}_n)\}$ such that

$$\begin{aligned} & (\delta_{n_1}, \mathbf{z}_{n_1}) \\ &= (\delta_1, \mathbf{z}_1) - ((\delta_1, \mathbf{z}_1)_{i_0} - \pi_{-1i_0}^{\mathcal{G}'}((\delta_1, \mathbf{z}_1)))\mathbf{e}_{i_0} \\ & \quad \vdots \\ & (\delta_{n_{l+1}}, \mathbf{z}_{n_{l+1}}) \\ &= (\delta_{n_l}, \mathbf{z}_{n_l}) - ((\delta_{n_l}, \mathbf{z}_{n_l})_{i_l} - \pi_{-1i_l}^{\mathcal{G}'}((\delta_{n_l}, \mathbf{z}_{n_l})))\mathbf{e}_{i_l}, \end{aligned} \quad (15)$$

where $1 < n_1 < n_2 < \dots < n_l < \dots$. $(\delta_{n_l}, \mathbf{z}_{n_l})_{i_l}$ denotes the i_l th element of vector $(\delta_{n_l}, \mathbf{z}_{n_l})$, where the indices i_l denote the only position in which $(\delta_{n_{l+1}}, \mathbf{z}_{n_{l+1}})$ differs from $(\delta_{n_l}, \mathbf{z}_{n_l})$. This subsequence can be thought of as the “off-springs” of vertex (δ_1, \mathbf{z}_1) through a series of projections, and they are not necessarily adjacent since there might be projections of other vertices happening in between. It can be shown that there is at least one such subsequence that has infinite length. With a slight abuse of notation, let $\{(\delta_{n_l}, \mathbf{z}_{n_l}), \forall l \geq 1\}$ denote such one subsequence. Since $(\delta_{n_l}, \mathbf{z}_{n_l}) \succeq \pi_{-1i_l}^{\mathcal{G}'}((\delta_{n_l}, \mathbf{z}_{n_l}))$, (15) together with the nonnegativeness of the vertex set \mathcal{T}_n implies that $(\delta_1, \mathbf{z}_1) \succeq (\delta_{n_1}, \mathbf{z}_{n_1}) \succeq \dots \succeq (\delta_{n_l}, \mathbf{z}_{n_l}) \succeq \dots \succeq \mathbf{0}$. Hence, $\lim_{l \rightarrow \infty} \|(\delta_{n_l}, \mathbf{z}_{n_l}) - (\delta_{n_{l+1}}, \mathbf{z}_{n_{l+1}})\| \rightarrow 0$. From (15) we know that $(\delta_{n_l}, \mathbf{z}_{n_l})$ and $(\delta_{n_{l+1}}, \mathbf{z}_{n_{l+1}})$ only differ in the i_l 's position. Thus

$$\begin{aligned} & \|(\delta_{n_l}, \mathbf{z}_{n_l}) - (\delta_{n_{l+1}}, \mathbf{z}_{n_{l+1}})\| \\ &= (\delta_{n_l}, \mathbf{z}_{n_l})_{i_l} - (\delta_{n_{l+1}}, \mathbf{z}_{n_{l+1}})_{i_l} \\ &= (\delta_{n_l}, \mathbf{z}_{n_l})_{i_l} - \pi_{-1i_l}^{\mathcal{G}'}((\delta_{n_l}, \mathbf{z}_{n_l})) \rightarrow 0 \text{ when } l \rightarrow \infty. \end{aligned} \quad (16)$$

Together with $\pi_{-1i_l}^{\mathcal{G}'}((\delta_{n_l}, \mathbf{z}_{n_l})) = \lambda_{n_l}((\delta_{n_l}, \mathbf{z}_{n_l}) + \mathbf{1}) - \mathbf{1}$ implying $\pi_{-1i_l}^{\mathcal{G}'}((\delta_{n_l}, \mathbf{z}_{n_l})) + \mathbf{1} = \lambda_{n_l}((\delta_{n_l}, \mathbf{z}_{n_l})_{i_l} + 1)$ and $(\delta_{n_l}, \mathbf{z}_{n_l}) \succ \mathbf{0}$ implying $(\delta_{n_l}, \mathbf{z}_{n_l})_{i_l} + 1 > 1$, it follows from (16) that $\lim_{l \rightarrow \infty} \lambda_{n_l} = 1$. That is,

$$\lim_{l \rightarrow \infty} (\delta_{n_l}, \mathbf{z}_{n_l}) \rightarrow \pi_{-1i_l}^{\mathcal{G}'}((\delta_{n_l}, \mathbf{z}_{n_l})) \quad (17)$$

TABLE II
COMPARISON OF PURE POWER CONTROL, ON-OFF SCHEDULING VERSUS JOINT POWER CONTROL AND SCHEDULING

Strategies (Algorithm)	Average Performance					
	$U_i(r_i) = \log(r_i)$		$U_i(r_i) = \frac{1}{1+\exp(-r_i+2)}$		$U_i(r_i) = \frac{1}{1+\exp(-r_i+4)}$	
	3-link	4-link	3-link	4-link	3-link	4-link
On-off Power Control without scheduling (Exhaustive Search)	4.4476	4.2951	2.6324	3.4315	1.7123	2.9151
Pure Power Control (MAPEL)	4.6801	5.9330	2.6750	3.4935	2.3273	2.9864
On-off Scheduling (Exhaustive Search)	5.1668	6.5933	2.8361	3.7103	2.3413	3.0047
Joint Power Control and Scheduling (A-S-MAPEL)	5.2276	6.7021	2.8450	3.7255	2.3752	3.0444

and

$$\lim_{l \rightarrow \infty} \Phi((\delta_{n_l}, z_{n_l})) \rightarrow \Phi(\pi_{-1}^{\mathcal{G}'}((\delta_{n_l}, z_{n_l}))). \quad (18)$$

Eqn. (17) implies that the subsequence $\{(\delta_{n_l}, z_{n_l})\}$ converges to the boundary of the region \mathcal{G}' where any point (δ, z) satisfies $(\delta, z) \succeq \mathbf{0}$. Since the nonnegative boundary of the region \mathcal{G}' is clearly equivalent to the upper boundary of the feasible region \mathcal{G} , (17) also implies that the subsequence $\{(\delta_{n_l}, z_{n_l})\}$ converges to the upper boundary of the feasible set \mathcal{G} . Since (δ_{n_l}, z_{n_l}) is a maximizer over the set \mathcal{S}_{n_l} , it is also the global optimum of Problem P_2 when the sequence converges. Since the CBS subsequence $\{z_{\delta'_{n_l}}\}$ satisfies $\Phi((\delta_{n_l}, z_{n_l})) \geq \Phi((\delta'_{n_l}, z'_{n_l}))$ for any n_l , it follows from (18) that

$$\begin{aligned} \lim_{l \rightarrow \infty} \Phi((\delta_{n_l}, z_{n_l})) \\ \rightarrow \Phi(\pi_{-1}^{\mathcal{G}'}((\delta_{n_l}, z_{n_l}))) \geq \lim_{l \rightarrow \infty} \Phi((\delta'_{n_l}, z'_{n_l})). \end{aligned} \quad (19)$$

On the other hand, according to the computation of $(\delta'_{n_l}, z'_{n_l})$, we can get

$$\lim_{l \rightarrow \infty} \Phi((\delta'_{n_l}, z'_{n_l})) \geq \Phi(\pi_{-1}^{\mathcal{G}'}((\delta_{n_l}, z_{n_l}))). \quad (20)$$

Together with $\pi_{-1}^{\mathcal{G}'}((\delta_{n_l}, z_{n_l}))$ being the global optimal of Problem P_2 when $l \rightarrow \infty$, eqns. (19) and (20) imply $(\delta'_{n_l}, z'_{n_l})$ is also the global optimum of Problem P_2 when $l \rightarrow \infty$. Note that the S-MAPEL algorithm terminates once the optimal solution to Problem P_2 is found. Therefore, the convergence of the subsequence $\{(\delta_{n_l}, z_{n_l})\}$ guarantees the convergence of the algorithm to the global optimal solution.

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