Hybrid Renewable Energy Investment in Microgrid

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***Abstract*—Both solar energy and wind energy are promising renewable sources to meet the world's problem of energy shortage in the near future. In this paper, we identify the complementary relation between solar power and wind power at certain locations of Hong Kong, and aim at studying the hybrid renewable energy investment in the microgrid. We jointly consider the investment and operation problem, and present a two-period stochastic programming model from the microgrid operator's perspective. In the first period, the operator makes optimal investment decisions on solar and wind power capacities. In the second period, the operator coordinates the power supply and demand in the microgrid to minimize the social operational cost. We design a decentralized algorithm for computing the optimal pricing and power consumption in the second period, and based on this solve the optimal investment problem in the first period. With realistic meteorological data obtained from Hong Kong observatory, we numerically demonstrate that the demand response saves 18% of the capital investment, and hybrid renewable energy investment reduces the generation capacity by up to 6.3% compared to a single renewable energy investment.**

I. INTRODUCTION

Aiming at reducing greenhouse gas emissions and enhancing the power grid reliability, many countries are building new power infrastructures known as the smart grid [1]. The major features of the smart grid include more distributed power generations (especially from renewable energy sources), two-way communications between the utility company and consumers for a better demand side management, and decentralized operations of power grid in the form of microgrids [1]. It's essential to understand the impact of these new features, and how to make optimal economic and technology decisions on the planning and operation of the smart grid.

Recently, there are many studies on power grid planning, integration of renewable energy, and demand response. Specifically, studies in [2] and [3] examined investment strategies on renewable energy generation through empirical (or numerical) approaches, without considering the power scheduling operation. Studies in [4] and [5] formulated the renewable generation investment optimization problems to meet inelastic demands, without considering consumers' demand responses. Another body of literature focused on the study of optimal demand response for residential consumers [6]–[8], data centers [9], and electric vehicles [10] in smart grid, by deriving the proper incentive schemes through either game theoretic models [6], [10] or optimization models [7]–[9]. The key idea of these studies is to design incentive mechanisms usually in the form of pricing schemes to users, and let cost-aware users schedule their elastic demands as response to the pricing over time. The study in [11] considered the impact of demand response on the thermal capacity investment, and showed that demand response reduces the thermal capacity investment.

To summarize, the existing literature focused on either renewable energy investment at a large time scale (years), or demand response optimization under given energy capacity at a small time scale (hours and days). However, the decisions at these two different time scales are actually tightly coupled. The renewable energy investment determines the (time varying) power supply availability, and thus affects the flexibility of users' demand response at a smaller time scale. Meanwhile, demand response can try to match the demand with the time varying renewable energy supply, and hence can maximize the benefit of renewable energy and even reduce unnecessary investment expenditure at a large time scale. In this paper, we will jointly consider the optimal renewable energy investment and demand response optimization in the smart grid.

In particular, this paper will focus on the hybrid renewable energy investment that involves both solar energy and wind energy, which are expected to be the most popular sources in the near future [12]. The optimal mix of solar and wind energy investment will highly depend on the stochastic nature of these two sources, which is highly location dependent. Hence we will rely on the meteorological data in Hong Kong to validate the practical relevance of our study, and examine the impacts of key system parameters and constraints on the optimal investment in renewable energy capacity.

Our goal is to develop a theoretical framework that captures the economic impact of renewable energy and demand response in the smart grid, and derive the optimal investment strategy and optimal demand response scheme based on realistic data. The main contributions of this paper are as follows.

- *Hybrid renewables modeling*: In Section II, based on meteorological data acquired from the Hong Kong Observatory, we identify the complementary relationship between solar power and wind power at certain locations of Hong Kong, and suggest hybrid renewable investment in both solar power and wind power.
- *Framework development*: In Section III, we develop a theoretical framework that enables us to derive the optimal hybrid renewable energy investment and operation in a demand responsive microgrid. The problem is challenging due to the coupling of decisions at different time scales.
- *Modeling and solution methods*: In Section IV, we formulate the joint investment and operation problems as a two-period stochastic program, design a distributed algorithm to achieve the optimal power scheduling in the second period, and derive a single level optimization formulation to solve the optimal investment in the renewable energy portfolio in the first period.

This work is supported by a grant from the Research Grants Council of the Hong Kong Special Administrative Region, China, under Theme-based Research Scheme through Project No. T23-407/13-N.

978-1-4799-4934-2/14/$31.00 ©2014 IEEE

- *Simulations and implications*: In Section V, numerical studies based on realistic meteorological data illustrate the optimal portfolio investment decisions, and demonstrate the benefits of jointly considering demand response and hybrid renewable energy technologies, in terms of reduced renewable energy investment.

II. Solar Power and Wind Power in Hong Kong

Both solar power and wind power are intermittent power sources, and their stochastic features are highly location dependent. Aiming at studying the renewable power patterns in Hong Kong, we acquire meteorological data from the Hong Kong Observatory. The data include the hourly solar radiation in King's Park (KP) of Hong Kong, and hourly wind speed at seven different locations of Hong Kong, as shown in Fig. 1. Since Hong Kong is relatively small, we assume that the solar radiation is the same across Hong Kong and is represented by the data measured in King's Park. The wind power, however, has clearly different patterns at different locations. In this paper, we will focus on one year meteorological data (from Sep. 1 2012 to Aug. 31 2013) to study the daily solar power and wind power productions. We find that the wind powers in five locations (KP, TMT, TPK, SHA, SKG) of Hong Kong have positive correlations[1] with solar power, while the correlation is negative in two locations (TC, WGL). As an example, in this paper we will focus on the measurement data in Tate's Cairn (TC), where solar and wind powers have a negative correlation.

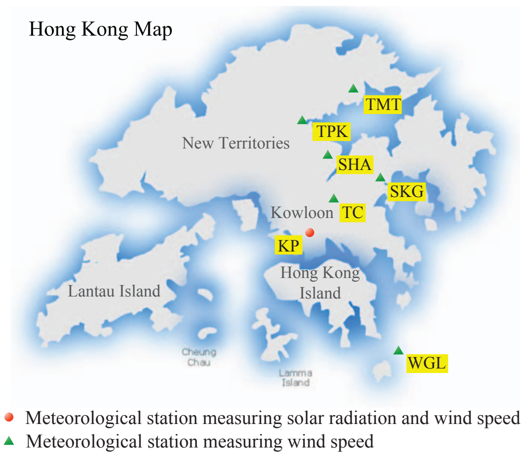

Fig. 1: Locations of meteorological stations

Based on the solar power model [13] and wind power model [14], we calculate the daily realization of hourly solar and wind power productions in 365 days based on the measurement of data of solar radiation and wind speed.[2] Each daily power production realization is called a scenario, and thus we obtain 365 scenarios for solar power and wind power respectively. We model the renewable energy generation scenarios as all the combinations of each solar power scenario and each wind power scenario, and the total number of renewable scenarios is 365×365. Due to the large number of scenarios which limits the computational tractability of the investment optimization problem, it is useful to choose a smaller subset of scenarios that can well approximate the original scenario set. Such technique has been widely used in economics and engineering research [15], [16] for the purpose of modeling stochastic processes.

We applied the forward scenario reduction algorithm [16] (details described in our online technical report [23]) to find a best scenario subset, and assign new probabilities to the smaller number scenarios. The key idea is to select a subset of scenarios to preserve, such that the corresponding reduced probability measure is the closest to the original measure. We set the number of preserved scenarios as 10, and generate selected scenarios for the solar power in KP (which we assume is the same as in TC) and wind power in TC, shown in Fig. 2. Therefore, for the renewable generation, we have a set of $10 \times 10 = 100$ scenarios denoted as Ω, and each renewable generation scenario $\omega \in \Omega$ has one solar power scenario and one wind power scenario.[3] Comparing (a) and (b), we can see that the solar power has a peak at noontime, while wind power is often adequate during night time. Therefore, solar power and wind power have a complementary relation, which motivates us to consider hybrid deployment of both. The data we use can be found at [24].

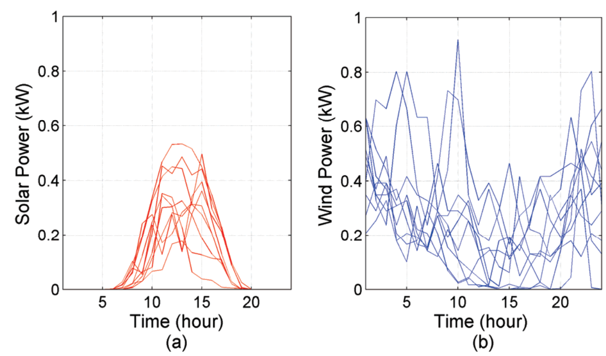

Fig. 2: Solar and wind power scenarios per 1kW capacity

III. System Model

Fig. 3 illustrates a typical microgrid, which consists of different sources of power supply and responsive demand. Within the microgrid, there is a local generation system, consisting of solar and wind renewable power generation. Besides the local energy generations, the microgrid is also connected to the main grid that provides power through conventional generation methods. The demand side consists of a set of electricity users $\mathcal{N} = \{1, ..., N\}$, and each user $i \in \mathcal{N}$ is equipped with a smart meter and energy scheduling module. The operator runs the microgrid, determines the investment in renewable energy capacities at a large time scale (years), as well as energy prices at a small time scale (hours and days). The users determine their energy consumptions based on the prices set by the operator (as the demand response).

From the operator's perspective, it needs to decide optimal renewable capacity investment and power scheduling, in which investment and scheduling decisions affect each other. We formulate the joint investment and operations problem as a stochastic program with two periods, as shown in Fig. 4. Specifically, the first-period problem is a long term renewable energy capacity investment problem, with the objective of min-

[1] We compute the correlations according to [12].

[2] The technical parameters of the solar power model and wind power model are shown in [23].

[3] For simplicity, we have assumed that the solar and wind power generations are independent. We will take the correlation between solar power scenario and wind power scenario as our future work to explore.

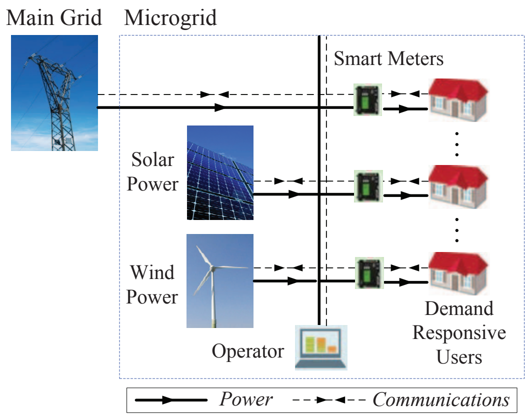

Fig. 3: System model

imizing the expected overall cost over an investment horizon $\mathcal{H} = \{1, ..., D\}$ of D days, subject to a budget constraint. An investment horizon usually corresponds to several years. The second-period problem is a power scheduling problem, with the objective of minimizing the social operational costs of both operator and users under a specific realization of renewable power generation within a smaller time window. The operational horizon is one day, which includes $\mathcal{T} = \{1, ..., T\}$ of T time slots (say 24 hours).

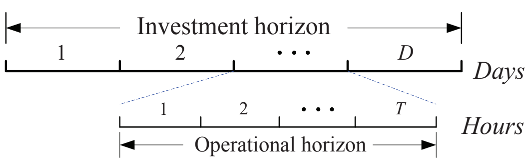

Fig. 4: Investment and operational horizons

Note that in the second period, the optimal power scheduling is affected by both power supply and demand, and the renewable power supply depends on the capacity investment decision made in the first period and the actual realized scenario. On the other hand, the expected overall cost in the first period includes both the one-time investment cost and the recurring social cost in all D days. Hence the decisions in two periods are coupled. In the following, we will first formulate the operation problem in the second period, and then formulate the investment problem in the first period.

A. Period 2: Microgrid operations

Microgrid operator acts as a social planner, with the objective of minimizing the social cost including both operator's cost and users' costs in an operational horizon. In this subsection, we first present the models of users and the operator, and then formulate the operator's social cost minimization problem.

1) User's model: We classify each user's load into two types: the flexible load and the inflexible load. The flexible load corresponds to the energy usage of those appliances such as electric vehicles, washing machines, and video game consoles, as a user may shift the flexible load over time without significant negative impact. The inflexible load corresponds to the energy usage of appliances such as lighting, refrigerators, and such load cannot be easily shifted over time. The demand response can only control the flexible load. We denote the corresponding decisions as $\boldsymbol{x}_i^\omega = \{x_i^{\omega,t}, \ \forall t \in \mathcal{T}\}$, where $x_i^{\omega,t}$ is user i's flexible load energy consumption in time slot $t \in \mathcal{T}$ under scenario ω.

The flexible load scheduling for all users needs to satisfy the following two constraints.

$$\underline{d}_i^t \le x_i^{\omega,t} \le \overline{d}_i^t, \ \forall t \in \mathcal{T}, \ i \in \mathcal{N}, \tag{1}$$

$$\sum_{t \in \mathcal{T}} x_i^{\omega,t} \le D_i, \ i \in \mathcal{N}. \tag{2}$$

Constraint (1) provides a minimum power consumption $\underline{d}_i^t$ and a maximum power consumption $\overline{d}_i^t$ for the user i in each time slot t. Constraint (2) corresponds to the maximum total flexible power demand D_i for user i in the entire operational horizon.

We further introduce a discomfort cost $C_i(\cdot)$, which measures user i's experience under $\boldsymbol{x}_i^\omega = \{x_i^{\omega,t}, \ \forall t \in \mathcal{T}\}$ which deviates from his preferred power consumption $\boldsymbol{y}_i = \{y_i^t, \ \forall t \in \mathcal{T}\}$ under a time-independent (and low enough) price. However, if the operator sets time-dependent prices, a user will schedule its flexible load to minimize the cost (more details in Section IV), i.e. shifting power consumption from high price time slots to low price time slots. The corresponding discomfort (or inconvenience) [17] is

$$C_i(\boldsymbol{x}_i^\omega) = \sum_{t \in \mathcal{T}} \left(x_i^{\omega,t} - y_i^t\right)^2. \tag{3}$$

2) Operator's model: We assume that the operator can predict the renewable energy production scenario ω accurately at the beginning of an operational horizon (a day).[4] In scenario ω and each time slot t, the operator determines the renewable power supply and the conventional power procurement to meet the total users' demand, which consists of the flexible power consumption $\boldsymbol{x}_i^\omega$ from each user $i \in \mathcal{N}$ and the aggregate inflexible load $\{b^t, \ t \in \mathcal{T}\}$ of all the users.

The renewable power supply $\boldsymbol{r}^\omega = \{r^{\omega,t}, \ \forall t \in \mathcal{T}\}$ and conventional power procurement $\boldsymbol{q}^\omega = \{q^{\omega,t}, \ \forall t \in \mathcal{T}\}$ should satisfy the following constraints.

$$0 \le r^{\omega,t} \le r_{max}^{\omega,t}, \forall t \in \mathcal{T}, \tag{4}$$

$$q^{\omega,t} \ge 0, \forall t \in \mathcal{T}, \tag{5}$$

$$r^{\omega,t} + q^{\omega,t} = b^t + \sum_{i \in \mathcal{N}} x_i^{\omega,t}, \ \forall t \in \mathcal{T}, \tag{6}$$

where $r_{max}^{\omega,t}$ in constraint (4) depends on the invested capacities of solar power α_s and wind power α_w, which are the operator's decision variables in the first period. For each unit of invested solar capacity and wind capacity, the corresponding solar power and wind power in scenario ω and time t will be $\boldsymbol{\eta}_s^\omega = \{\eta_s^{\omega,t}, \forall t \in \mathcal{T}\}$ and $\boldsymbol{\eta}_w^\omega = \{\eta_w^{\omega,t}, \forall t \in \mathcal{T}\}$, respectively. Hence we have $r_{max}^{\omega,t} = \eta_s^{\omega,t}\alpha_s + \eta_w^{\omega,t}\alpha_w$. Constraint (5) means that the operator can only purchase power from the main grid, but cannot sell power to the main grid, assuming that the main grid does not accommodate distributed generations in the microgrid. We assume that the main grid has adequate power to meet the demand of the microgrid, hence there is no upper-bound of $q^{\omega,t}$ in (5). Constraint (6) is the power balance constraint between supply and demand in time slot t.

Different from the conventional power generation, the renewable power generation does not consume fuel sources, so we assume zero cost of generating renewable power. Therefore,

[4] The short-run (day-ahead) renewable energy forecast can be quite accurate in practice [18], [19]. We will consider the impact of prediction error in the future work.

the operator will try to use as much renewable power as possible to meet the demand. We let $\boldsymbol{Q}^{\omega} = \{Q^{\omega,t},\ t \in \mathcal{T}\}$ denote the aggregate supply, i.e., $Q^{\omega,t} = r^{\omega,t} + q^{\omega,t}$, and rewrite the power balance constraint (6) as follows:

$$Q^{\omega,t} = b^t + \sum_{i \in \mathcal{N}} x_i^{\omega,t},\ \forall t \in \mathcal{T}. \tag{7}$$

If the operator has enough renewable power to meet the aggregate demand, i.e. $r_{max}^{\omega,t} \geq Q^{\omega,t}$, then there is no need to purchase any conventional power, i.e. $q^{\omega,t} = 0$. On the other hand, if $r_{max}^{\omega,t} < Q^{\omega,t}$, the operator will first use all the renewable power $r^{\omega,t} = r_{max}^{\omega,t}$, and then purchase conventional power $q^{\omega,t} = Q^{\omega,t} - r_{max}^{\omega,t}$ to meet the power deficit. The production cost of conventional power has a quadratic form [20], and thus we define the operator's cost as

$$C_o(\boldsymbol{Q}^{\omega}) = \sum_{t \in \mathcal{T}} \left[\left(Q^{\omega,t} - \eta_s^{\omega,t}\alpha_s - \eta_w^{\omega,t}\alpha_w \right)^+ \right]^2, \tag{8}$$

where $(z)^+ = \max\{z, 0\}$ for any value z.

3) The second-period problem: Next we state the second period problem, where the operator coordinates aggregate power supply $\boldsymbol{Q}^{\omega}$ and schedules users' power consumptions $\boldsymbol{x}_i^{\omega}$ to minimize the social cost consisting of the operator's cost $C_o(\boldsymbol{Q}^{\omega})$ and all the users' costs $C_i(\boldsymbol{x}_i^{\omega})$ as follows.

Social cost minimization problem in the second period P2:

$$\min_{\boldsymbol{Q}^{\omega}, \boldsymbol{x}_i^{\omega}} \ \beta C_o(\boldsymbol{Q}^{\omega}) + (1-\beta) \sum_{i \in \mathcal{N}} C_i(\boldsymbol{x}_i^{\omega}) \tag{9}$$
$$s.t. \quad (1), (2), (7).$$

The operator adopts the same parameter β selected by individual users in (16) to tradeoff the costs for the operator and users. Problem **P2** is convex, and can be solved efficiently if a centralized optimization is possible. However, this may not be feasible in practice, as the operator cannot directly control users' power consumptions. We will discuss the design of a decentralized algorithm in Section IV.

B. Period 1: Hybrid renewable energy investment

In the first investment period, the operator needs to determine the capacities of the solar power and wind power facilities (α_s and α_w) for the entire investment horizon, subject to a budget constraint B. These capacity decisions will determine the renewable power production in each day of the second period, and consequently affect the power scheduling and operational cost. The operator wants to make optimal investment decisions to minimize the overall cost, including both the capital investment and the expected operational cost in the second period.

The capital investment cost can be represented as

$$C_I(\alpha_s, \alpha_w) = c_s\alpha_s + c_w\alpha_w, \tag{10}$$

where c_s and c_w denote the investment costs of solar power and wind power per kW, respectively. The investment cost covers all expenditures, e.g., installation and maintenance of photovoltaic panel for solar energy, turbine for wind energy, controllers, inverters, and cables.

The expected operational cost $\mathbb{E}_{\omega}[f(\cdot)]$ is a function of the invested capacities α_s and α_w,

$$\mathbb{E}_{\omega \in \Omega}\left[f(\alpha_s, \alpha_w, \omega)\right] = \sum_{\omega \in \Omega} \pi_{\omega} \left[f(\alpha_s, \alpha_w, \omega)\right], \tag{11}$$

where $\omega \in \Omega$ denotes the renewable power scenario with a realization probability π_{ω}, which is obtained by the scenario reduction algorithm in Section II. Specifically, the operational cost function in scenario ω is the optimized second-period objective value (i.e. minimized social cost) in scenario ω:

$$f(\alpha_s, \alpha_w, \omega) = \min_{\boldsymbol{Q}^{\omega}, \boldsymbol{x}_i^{\omega}} \left(\beta C_o(\boldsymbol{Q}^{\omega}) + (1-\beta) \sum_{i \in \mathcal{N}} C_i(\boldsymbol{x}_i^{\omega}) \right), \tag{12}$$

where the cost depends on the renewable power supply in scenario ω and users' demand responses in the second period.

The first-period optimization problem is subject to a budget constraint $c_s\alpha_s + c_w\alpha_w \leq B$, and the solar power and wind power capacities must be non-negative. To summarize, the first-period problem is as follows.

The first-period problem P1:

$$\min_{\alpha_s, \alpha_w} \quad \delta C_I(\alpha_s, \alpha_w) + (1-\delta) D \cdot \mathbb{E}_{\omega \in \Omega}\left[f(\alpha_s, \alpha_w, \omega)\right] \tag{13}$$
$$s.t. \quad c_s\alpha_s + c_w\alpha_w \leq B, \tag{14}$$
$$\alpha_s \geq 0,\ \alpha_w \geq 0, \tag{15}$$

where the objective function consists of capital investment cost C_I and expected operational cost $\mathbb{E}_{\omega}[f(\cdot)]$ over all scenarios $\omega \in \Omega$. The weight parameter $\delta \in [0,1]$ trades off the long-term investment cost and the short-term operational cost, and D is the number of days over the investment horizon.

IV. Solution Method

To solve the above two-period stochastic programming problem, we start with the second-period problem **P2** and solve it using a distributed algorithm, and then solve the first-period problem **P1** to obtain the optimal renewable energy investment.

A. Period 2: Optimal power scheduling

As mentioned in Section III, it is not practical for the operator to solve **P2** centrally and control users' power consumptions directly. Instead, the operator can set day-ahead prices $\boldsymbol{p}^{\omega} = \{p^{\omega,t},\ t \in \mathcal{T}\}$ for the users in scenario ω, and let users choose the proper power scheduling accordingly. In the following, we first present a user's total cost minimization problem given the prices. Then we discuss the operator's optimal choices of prices so that the users' power scheduling decisions coincide with the optimal solution of Problem **P2**.

1) User's problem: In scenario ω, user i receives the price signals $\boldsymbol{p}^{\omega}$ and schedules the power consumption $\boldsymbol{x}_i^{\omega}$ to minimize the total cost. Specifically, user i's total cost consists of two parts: energy cost C_e and discomfort cost C_i. The energy cost[5] of user i depends on the price and user i's power consumption, which can be represented as $C_e(\boldsymbol{x}_i^{\omega}) = \sum_{t \in \mathcal{T}} p^{\omega,t} x_i^{\omega,t}$. Therefore, we have the following total cost minimization problem for user i in scenario ω.

User's total cost minimization problem Pu:

$$\min_{\boldsymbol{x}_i^{\omega}} \ \beta C_e(\boldsymbol{x}_i^{\omega}) + (1-\beta) C_i(\boldsymbol{x}_i^{\omega}) \tag{16}$$
$$s.t. \quad (1), (2),$$

where the users choose β to tradeoff the energy cost and user's discomfort cost. The operator chooses the same β in the social cost minimization problem **P2**.

[5]The user cannot change the energy cost related to inflexible load, hence we do not consider that in user's optimization problem.

2) Optimal pricing and decentralized algorithm: We denote $\boldsymbol{p}^{\omega *} = \{p^{\omega,t*}, \forall t \in \mathcal{T}\}$ as the optimal pricing that induces the socially optimal power consumption $\{\boldsymbol{x}_i^{\omega *}, \forall i \in \mathcal{N}\}$ in scenario ω (i.e. the optimal solution of Problem **P2**). We have the following theorem.

Theorem 1: In each scenario ω, the optimal pricing scheme $\boldsymbol{p}^{\omega *}$ that induces the socially optimal power consumptions $\boldsymbol{x}_i^{\omega *}$ for each user i satisfies the following relationship,

$$p^{\omega,t*} = \begin{cases} \frac{\partial C_o(\boldsymbol{Q}^\omega)}{\partial Q^{\omega,t}}\Big|_{Q^{\omega,t}=Q^{\omega,t,*}}, & \text{when } Q^{\omega,t*} > r_{max}^{\omega,t}, \\ 0, & \text{when } Q^{\omega,t*} \leq r_{max}^{\omega,t}. \end{cases} \quad (17)$$

Theorem 1 motivates us to design a decentralized algorithm in **Algorithm 1**, where the operator sets the day-ahead prices and users respond to the prices by determining their power consumptions. At the beginning of each day, the operator and each user's smart power control module compute the electricity prices and power consumptions iteratively.

Algorithm 1 Decentralized algorithm in the microgrid

1: **Initialization**: iteration index $k = 0$, error tolerance $\epsilon > 0$, stepsize $\gamma > 0$, predicted scenario ω, and $x_i^{\omega,t}(0) = y_i^t$.
2: **repeat**
3: **Operator**: At the k-th iteration, the operator collects each user's consumption and computes the aggregate supply $\boldsymbol{Q}^\omega$, and sets the prices $p^{\omega,t}(k)$ according to (17).
4: **Users**: Each user i solves Problem **Pu** by updating the power consumption $x_i^{\omega,t}(k+1)$ based on the price $p^{\omega,t}(k)$:

$$\hat{x}_i^{\omega,t}(k+1) = x_i^{\omega,t}(k) - \gamma(k)\left((1-\beta)\frac{\partial C_i(\boldsymbol{x_i}^\omega(k))}{\partial x_i^{\omega,t}(k)} + \beta p^{\omega,t}(k)\right).$$

5: Project the power consumption on the feasible set by solving the following problem:

$$\min_{\boldsymbol{x}_i^\omega(k+1)} \quad \| \boldsymbol{x}_i^\omega(k+1) - \hat{\boldsymbol{x}}_i^\omega(k+1) \| \qquad s.t. \quad (1), (2).$$

6: $k = k+1$;
7: **until** $\| \boldsymbol{p}^\omega(k) - \boldsymbol{p}^\omega(k-1) \| \leq \epsilon$.
8: **end**

We can see that the decentralized **Algorithm 1** requires minimum information exchange between the operator and users. The users only report their power consumptions to the operator, and the operator broadcasts the prices to all users based on the aggregate power load. There is no need for the users to directly coordinate with each other or to reveal their private information such as cost function and consumption constraints. The prices set by the operator are the same for all users, and reflect the total power load without disclosing individual user's power consumption.

Theorem 2: **Algorithm 1** is a sub-gradient projection algorithm for solving **P2**, and with a diminishing stepsize it converges to the socially optimal price and power consumption $\{\boldsymbol{p}^{\omega *}, \boldsymbol{x}^{\omega *}\}$ for each ω.

Note that the diminishing stepsize satisfies the following conditions: $\lim_{k\to\infty} \gamma(k) = 0$ and $\lim_{k\to\infty} \sum_k \gamma(k) = \infty$.

B. Period 1: Optimal hybrid renewable energy investment

After solving the second-period problem **P2**, we solve the first-period problem **P1** as follows.

Theorem 3: The operator's investment problem **P1** is equivalent to the following single optimization problem **EP1**.

$$\min_{\alpha_s, \alpha_w, \boldsymbol{Q}^\omega, \boldsymbol{x}^\omega} \delta\left[c_s\alpha_s + c_w\alpha_w\right] + (1-\delta)D\cdot\mathbb{E}_\omega\Big[\beta C_o(\boldsymbol{Q}^\omega) + (1-\beta)\sum_{i\in\mathcal{N}} C_i(\boldsymbol{x}_i^\omega)\Big],$$
$$s.t. \quad (1), (2), (7), (14), (15),$$

which is a convex quadratic program and can be solved using a standard interior-point method [21]. We assume that the operator can estimate the information about user's power consumption behaviors through a survey or daily operations, and the parameters in (1) and (2) are known to the operator. Then the operator solves the equivalent problem **EP1** for optimal investment in a centralized manner. For the proofs of Theorems, please refer to [23].

V. Simulation Results

We assume that the investment horizon includes $D = 3650$ days (10 years). We use the load curve in [22] as the preferred power consumption, and set $\delta = 0.1$ and $\beta = 0.5$. The investment costs of solar power and wind power per kW are set as $c_s = 16000$ HKDs and $c_w = 5500$ HKDs [3]. We obtain renewable power scenarios as discussed in Section II, with details described in our technical report [23].

A. Benefit of demand response

First, we study the benefit of adopting the incentive-based demand side management scheme. The result is shown in Fig. 5, where we assume that the investment budget in the first period is 5 million HKDs. Without incentives (hence keeping the prices low and the same for 24 hours), a user i will choose the power consumption according to $\boldsymbol{y}_i$. In that case, simulation shows that it is optimal for the operator to use the entire 5 million HKDs budget during the first period. With incentives, the operator sets day-ahead prices so that to steer the users' power consumptions to the socially optimal values. The total optimal investment expenditure reduces to 4.1 million HKDs, which implies that the demand response may significantly reduce the system cost and avoid over-investment.

B. Benefit of hybrid renewable investment

We compare the optimal hybrid renewable capacities with the cases where the operator only invests in a single renewable source. Fig. 6 shows the optimal capacities in three cases: solar alone, wind alone, and hybrid renewable sources. The red bar represents the capacity invested in the solar power, and the blue bar represents the capacity invested in the wind power. We can see that the hybrid renewable solution has the smallest total capacity 549 kW, with 99 kW in solar power and 450 kW in wind power. Compared with the single renewable energy investment, the hybrid renewable energy investment saves up to 37 kW renewable generation capacity. The results demonstrate the benefit of the hybrid renewable investment in terms of reducing the need of the total generation capacity.

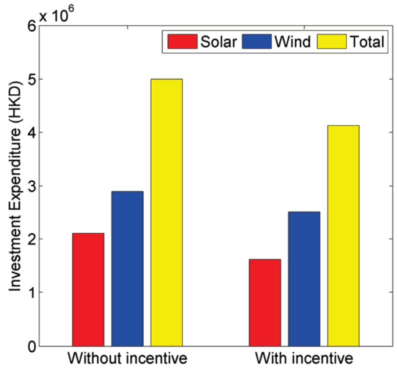

Fig. 5: Investment expenditure

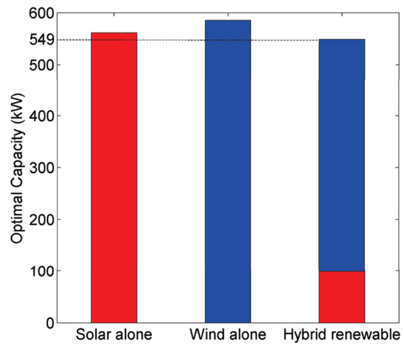

Fig. 6: Optimal capacity investment

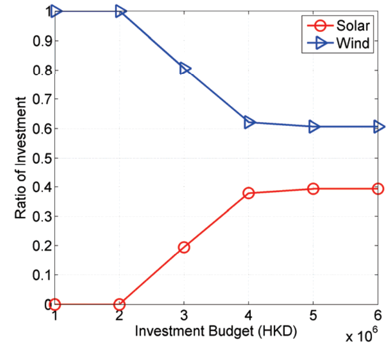

Fig. 7: Optimal ratio of investment in solar and wind power

C. Impact of budget

We study the impact of budget on the optimal ratio of investment in solar power and wind power, as shown in Fig. 7. When the budget is tight (as 1 million HKDs), the investment priority is wind power because it's more economical. While the budget increases, the investment prefers more solar power, because the solar power fits the daily demand better (i.e., more energy use during the day) even though it is more expensive.

VI. Conclusion

We propose a theoretical framework to study the joint investment and operation problem with renewable energy in the microgrid. With realistic meteorological data, our model provides the optimal investment decisions on hybrid renewable powers and optimal daily operation (pricing and power scheduling) of the microgrid. The simulation studies demonstrate the economic benefit of demand response and hybrid renewable investment. In the future, we will further consider the investment problem at different locations with different wind power scenarios, and impacts of key system parameters on optimal investment, operation and power scheduling.

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